Phys 449 Exam 1.

1) At $t = 0$ an electron in a magnetic field $B$ is prepared in the $|+\rangle$ state.
   a. What frequency should an additional oscillating field be applied to cause Rabi oscillations of the electron?
   b. A Rabi pulse is applied to the electron to put it in the state $|+\rangle - i|\rangle$ at $t = t_1$. Find $\langle S_z \rangle$ as a function of time $t > t_1$ after the Rabi pulse is turned off.

2) An atom is in the $^3D_1$ state with $m_j = 1$.
   a. Give the values of $s, l,$ and $j$ for this state.
   b. Write out the angular momentum wavefunction for this state in the $|m_lm_s\rangle$ basis.
   c. What is $\langle S_z \rangle$ in this state?

3) A free electron moves in a magnetic field $B = B\hat{z}$. Find the energy levels in terms of appropriate quantum numbers and fundamental constants.

4) A deuterium molecule consists of 2 electrons and 2 deuterons (the deuteron has nuclear spin $i = 1$). Its lowest energy states have total electron spin $s = 0$, but the molecule can also rotate, with rotational angular momentum quantum numbers $n = 0,1,\ldots$ The resulting wavefunction for the molecule is a product of the rotational wavefunction $|n m_n\rangle = Y_{nm_n}(\theta, \phi)$ and the appropriately symmetrized nuclear spin wavefunction.
   a. What are the possible total nuclear spin quantum numbers for the molecule?
   b. For each of your answers to a., give the exchange symmetry of the nuclear spin wavefunction. Hint: use your Clebsch-Gordan table.
   c. Given that the rotational wavefunctions are multiplied by $(-1)^n$ upon exchange of the two nuclei, what must the nuclear spin quantum number be for the states with $n$ odd?
5) (Take home) A AgHe molecule has a single electron, a nucleus of spin \( i = 1/2 \), and obeys the Hamiltonian \( H = \hbar b \mathbf{N} \cdot \mathbf{N} + \hbar a \mathbf{I} \cdot \mathbf{S} + g_s \mu_B B S_z \). Plot the energies as a function of magnetic field for \( b = 6 \) GHz and \( a = -1.7 \) GHz, \( n = 0,1,2 \). Note on your plot the places where states of different \( n \) become degenerate.