In this lecture I continue with examples.

**Time Dilation**

**Consider:**

\[
\begin{align*}
x' &= y (x - \beta c t) \\
ct' &= x (ct - \beta x) \\
\end{align*}
\]

\[
\beta = \frac{v}{c} \\
\gamma = \left[1 - \beta^2\right]^{-1/2}
\]

Suppose both clocks are coincident when \(x = x' = 0\) and \(t = 0\).

**This means that from O' - the moving coordinate system:**

The clock in O looks slow because \(c dt' = \gamma c dt\).

And from coordinate system O - in which O is moving:

The clock in O' looks slow since \(c dt = \gamma c dt'\).

The equations look inconsistent because in its rest frame the clock is at the origin. If each coordinate frame had a muon at rest, each frame would say that the moving muon lives longer. So at rest the unstable muon (and other unstable particles) behaves as:

\[
\mu^+ \rightarrow e^+ \bar{\nu}_e \nu_e \quad \tau_{\mu} = 2.2 \times 10^{-6} \text{ sec c.}
\]

\[
dN = -\frac{dN}{dt} = \frac{dN(t)}{d\tau_{\mu}} \rightarrow N(t) = N(0) e^{-t/\tau_{\mu}} \quad \text{exponential decay}
\]

**Decay length - non-relativistic:**

\[
L = \nu \tau_{\mu} = \beta c \tau_{\mu}
\]

CT for muon = 660 meter

**With time dilation, relativistic decay length:**

\[
L = \beta \gamma c \tau_{\mu}
\]

The factor \(\gamma = \frac{E}{mc^2}\) increases the decay length for relativistic particles. As we discussed, a 10 GeV muon has \(\gamma \approx 100\) \(\Rightarrow 10 c \tau_{\mu} \approx 66\) km, so a majority of muons created ~20 km in the atmosphere reach the surface.
Length contraction

If a muon to the moon:
\[ d = 240,000 \text{ miles} \]
\[ c = 186,000 \text{ miles/second} \]
\[ \frac{d}{c} = \text{ EARTH} \quad \text{MOON} \]

\[ \Rightarrow \text{ light takes } \sim 1.3 \text{ second to cover distance. The muon} \]
\[ \text{lifetime at rest is } 2 \times 10^{-6} \text{ sec. We need the muon to live} \]
\[ \sim 1.3 \text{ sec. } \Rightarrow \frac{2}{1.3} = \frac{7 \times 10^5}{2 \times 10^{-6}} \Rightarrow E = 7 \times 10^3 \text{ eV} = 70 \text{ keV} \]

This is currently beyond our technology, but present in cosmic ray showers, we can safely set \( \beta = 1 \).

Now consider the muon frame of reference. The moon is coming at you, with a speed virtually that of light, but the trip only takes 2 ysec.

The velocity is the same, and the time is shorter, so the distance must be shorter by the same factor. In the muon reference frame:

\[ d' = \frac{d}{y} \]

which is length contraction.

There is an alternative derivation:

Consider

\[ y \quad | \quad y' \quad | \quad v \]
\[ 0 \quad \langle \langle x \rangle \rangle \quad 0' \quad \langle \langle x' \rangle \rangle \]

Let a length \( l \) lie along the \( x \)-axis in frame 0. To measure the length in \( 0' \), the signals must reach the moving frame at the same time \( t' \). For example, think of a photograph.

\[ x' = y(x - \beta ct) \]
\[ ct' = y(ct - \beta x) \]

measure the origin when the \( y \)-coordinate system coincide at \( t = t' = x = x' = 0 \)
For \( x = l \) \( \gamma = 0 = ct - \beta \gamma \) \( \Rightarrow \) \( ct = \beta \gamma \)

\( \Rightarrow l' = \gamma (l - \beta^2 \gamma) = \frac{l}{\gamma} \)

This is also length contraction.