I review four optics phenomena and then connect them to special relativity. I thank Professor Lee Pondrom for his insights and for his lecture notes, which form the basis of these notes.

(1) Aberration of starlight
   
   (a) What is it?

   A star perpendicular to Earth's orbit and lying along the axis of the orbit appears to move around in a small circle as the Earth orbits the Sun.

   Let \( v = \text{speed of Earth in its orbit w.r.t. the Sun} \). Suppose there is a telescope pointing at the star. At time \( t = 0 \) light enters the top of the telescope, and strikes the bottom at time \( t = L \cos \alpha / c \).

   In that time the bottom of the telescope has moved a distance \( v t = L \sin \alpha \).

   \[
   \Rightarrow \quad \frac{v}{c} = \tan \alpha = \frac{\alpha}{10 \text{ radian}} = 0.1 \text{ mrad}
   \]

   Astronomers use degrees, minutes, and seconds of arc, like a clock. 1 second of arc \( = 4.87 \times 10^{-6} \text{ radian} \).

   \[
   \Rightarrow \quad \alpha = 20.5 \text{ second of arc}.
   \]

   This angle follows you around the Earth's orbit. The star seems to move in a small circle. If you know \( v \), which we do, by measuring \( d \) we get the speed of light \( c \).

   Also, since the speed of light \( c \) is taken the same regardless of where you are in the orbit, and the star seems to move in a circle, aberration of starlight was used as evidence against aether theory.
2) Fresnel Drag

Question: Does light go faster in a moving medium?
Answer: Yes. The surprise lies in how fast.

Suppose that water moves at a speed \( v \) along the direction of a beam of light. The speed of light in the water is:

\[ u = \frac{c}{n} + v \left(1 - \frac{1}{n^2}\right) \]

where \( n \) is the index of refraction \( (n = 1.33 \text{ for water}) \). \( \left(1 - \frac{1}{n^2}\right) \) is called the Fresnel drag coefficient and vanishes if \( n = 1 \). Let us derive this using Einstein's velocity addition theorem:

\[ \begin{align*}
    y' &\rightarrow \ y \\
    0' &\rightarrow \ 0 \\
    0 \rightarrow \ 0' \\
\end{align*} \]

In \( O' \), light travels with \( u' = \frac{c}{n} \) (the water).

In \( O \):

\[ u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{\frac{c}{n} + v}{1 + \frac{v}{n^2}} \]

\[ = \frac{c}{n} + v \left(1 - \frac{1}{n^2}\right) \]

to \( 1 \text{st order in } (v) \).

What follows?
* \( n = 1 \) in vacuum, so there is no Fresnel drag in vacuum.
* This provides strong evidence that Michelson & Morley would see nothing.

3) FIZEAU'S EXPERIMENT

The speed of light in and back:

\[ \begin{align*}
    \text{Out: } u_1 &= \frac{c}{n} + v \left(1 - \frac{1}{n^2}\right) \\
    \text{Back: } u_2 &= \frac{c}{n} - v \left(1 - \frac{1}{n^2}\right) < u_1
\end{align*} \]
What are the times out and back?

Out: \( t_1 = \frac{L}{u_1} \)

Back: \( t_2 = \frac{L}{u_2} \)

Time difference \( \Delta t = t_2 - t_1 = L \left( \frac{u_2 - u_1}{u_1 u_2} \right) = \frac{2vL(1-\frac{1}{n^2})}{c/n^2} \)

to first order in \((v/c)\)

\( \Delta t = \frac{2vL}{c^2} (n^2-1) \)

Phase difference \( \Delta \phi = \left( \frac{2\pi c}{\lambda} \right) \Delta L = 4\pi \left( \frac{v}{c} \right) \left( \frac{L}{\lambda^2} \right) (n^2-1) \)

to first order in \((v/c)\).

Put in numbers: \( v = 10 \text{ m/s} \) \( \Rightarrow \frac{v}{c} = \frac{1}{3} \times 10^{-7} \)

\( L = 2 \text{ meter}, \ \lambda = 6 \times 10^{-7} \text{ m} \)

\( \Rightarrow \Delta \phi = 4\pi \times \left( \frac{1}{3} \times 10^{-7} \right) \left( \frac{1}{3} \times 10^{-7} \right) = 1.4 \text{ radians} \)

which is easily observable, and confirmed by experiment.

Note: If \( n=1 \) (vacuum) there is no effect — another clue that the Michelson & Morley experiment would see nothing.

(4) Sagnac's experiment

For many years, this experiment formed the basis for optical gyroscopes used, e.g., in aircraft and other inertia guidance systems.

Suppose a light source and detector are mounted on the rim of an air light pipe in the shape of a circle of radius \( R \). The entire device rotates at an angular speed \( \omega \). The source emits light going both ways around the ring. The light goes around the ring in a time \( t = 2\pi R/\omega \). The time is measured in the rest frame to avoid confusion due to time dilation.
In this time the source & detector have both moved a distance:

$$R \omega t = \left( \frac{2 \pi R^2}{c} \right) \omega$$

So the difference in length that the two light signals travel to reach the detector is:

$$\Delta L = \frac{4 \pi R^2 \omega}{c} = \left( \frac{4 \pi A}{c} \right) \omega$$

$$A = \text{area} = \pi R^2$$

This corresponds to a time difference:

$$\Delta t = \frac{4 \pi A \omega}{c} \quad \text{or a phase difference} \quad \Delta \phi = \left( \frac{8 \pi A}{\lambda c} \right) \omega$$

If you put in reasonable numbers you get measurable phase shifts.

The phase shift formula holds for any shape closed area $A$. The Michelson–Morley experiment could not detect, but did not rule out, any area.

If an optical fiber with refractive index $(n)$ is used:

$$\Delta t = \left( \frac{4 A \omega}{c^2} \right) n^2$$

However, in this case there will also be the Fresnel drag coefficient, which goes the other way:

$$\Delta t_f = \left( \frac{4 A \omega}{c^2} \right) (n^2 - 1)$$

So the net time difference is:

$$\Delta t_{net} = \left( \frac{4 A \omega}{c^2} \right) \omega$$

the same as for air. So the index of refraction plays no role in the Sagnac effect.

Now let me provide a summary of special relativity,
\[ x' = \gamma (x - \beta ct) \quad y = \frac{1}{\sqrt{1 - \beta^2}} \quad \beta = \frac{v}{c} \]
\[ ct' = \gamma (ct - \beta x) \]

Inverse transformation: all we do is change \( \beta \to -\beta \)
\[ x = \gamma (x' + \beta ct') \quad c^2 t'^2 - x'^2 = c^2 t^2 - x^2 \]
\[ y' = y \]
\[ c t = \gamma (ct' + \beta x') \]

Velocity addition:
\[ \left( \frac{1}{c} \right) u_x' = \left( \frac{u_x}{c} \right) - \beta \]
\[ 1 - \left( \frac{\beta u_x}{c} \right) \]

For two velocities \( \beta_1 \) and \( \beta_2 \):
\[ 0 \xrightarrow{\beta_1} \frac{0}{\beta_2} \]
\[ \beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \]

Remember, while \( y' = y \), \( u_y' \neq u_y \) because \( t' \neq t \):
\[ \left( \frac{u_y'}{c} \right) = \left( \frac{u_y}{c} \right) \quad \text{Note: } u_x \text{, not } u_y \text{, appears in denominator} \]

Time dilation - when clocks start at the same origin:
\[ ct' = \gamma (ct - \beta x) \quad \text{When } x = 0: \ ct' = \gamma ct \]

or a time interval is: \( c dt' = \gamma c dt \)
\[ \Rightarrow \text{O' says O clocks run slower. But, } ct = \gamma ct' \text{, too, so O says O' clock runs slower. The moving clock runs slower.} \]

Example: Muons in cosmic rays. The muon is unstable:
\[ \mu^+ \to e^+ + \nu_e + \bar{\nu}_e \]
\[ -16- \]
Which in its rest frame has a lifetime \( \tau_y = 2.2 \times 10^{-6} \) sec.

For muons at rest: \( N(t) = \text{number of muons left after time } t \)  
\[ = N_0 e^{-t/\tau_y} \]  
 exponential decay.

Cosmic ray protons produce muons in the upper atmosphere about 20 km up. A high energy muon travels at almost the speed of light:
\[ c \tau_y = 660 \text{ meters}. \]

If there was no relativistic time dilation, only:
\[ e^{-20\times 660} \approx e^{-30} \approx 0 \] no muons would survive to reach the Earth's surface. However, muons do reach the surface. The correct decay length is:
\[ \gamma c \tau_y \]

We will soon learn that \( \gamma = \frac{E}{m c^2} \), where \( m = \text{rest mass of muon} \)
\[ m c^2 = 106 \times 10^6 \text{ eV} = 106 \text{ MeV} \] and \( E \) is the total energy of the muon, typically \( 10^{10} \text{ eV} = 10 \text{ GeV} \).

\[ \Rightarrow \gamma \approx 100 \Rightarrow \text{Muon decay length } x = \gamma c \tau_y \approx 66 \text{ km} \]