The Michelson - Morley experiment is based on the view, for which there was very strong evidence, that light was a wave. To 19th century scientists and engineers, a wave needed a medium to travel in - hence the "AETHER" and Michelson & Morley attempt to measure the speed of the Earth w.r.t. the aether medium. The idea is very similar to the swimmer (the Earth) and the creek water (the aether medium).

People knew that light travels in vacuum with a speed \( c = 3,000,000 \text{ m/s} \). They also knew that light is composed of an electric + magnetic field vectors, both vectors perpendicular to each other and to the direction of motion. So light was described by an electric polarization vector \( \mathbf{E}(\mathbf{r}, t) \), a frequency \( f \), a wavelength \( \lambda \), a speed \( c = f \lambda \) and a wavevector

\[
\mathbf{k} = \frac{2\pi}{\lambda} \quad \mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \cos (\mathbf{k}\cdot\mathbf{r} - \omega t)
\]

\( \mathbf{k}\cdot\mathbf{r} - \omega t = \) phase \( \phi \), \( \mathbf{E}_0 = \) amplitude, \( \mathbf{e} = \) direction of polarization

Phase \( \phi = \mathbf{k}\cdot\mathbf{r} - \omega t = \) constant travels at the speed of light. Suppose \( \mathbf{k} \parallel \mathbf{e} \Rightarrow KZ - \omega t = \) constant \( \Rightarrow \frac{dz}{dt} = \omega = c \).

Now suppose that aether is at rest w.r.t. the Sun. The Earth moves around the Sun at a speed \( v \approx 3 \times 10^4 \text{ m/s} \), so \( v < c \).

The goal of Michelson & Morley was to detect this (small) motion of the Earth - the observer - w.r.t. the Sun - the source. The basic instrument looks like:

```
A  BEAM SPLITTER  B
SOURCE  DETECTOR
```
The beam splitter cause \( \frac{\pi}{2} \) the light to go to mirror \( A \), 
\( \frac{\pi}{2} \) to mirror \( B \). The optical lengths \( BS-A \) and \( BS-B \) are 
carefully made the same. If the instrument is perfectly 
aligned, there is complete destructive interference. In 
reality, there is a (small) angle between \( B \) and \( A \). 
This small misalignment leads to bright and dark lines - 
FRINGES - in the field of view.

The distance between two bright (or dark) fringes, equals 
to \( (2\pi) \) radian or \( 360^\circ \) in phase. Changing 
either optical 
path causes the fringes to move.

Now suppose the interferometer is moving on Earth w.r.t. the 
aether: 
\[
\begin{align*}
\vec{V} &= \text{speed of aether}, \\
v &= \text{speed of aether}, \\
\end{align*}
\]

The legs \( OA \) and \( OB \) are the same as our swimmer in lecture 
one, except that the "swimmer" speed = \( c \) and \( (v) \) is the 
speed of the aether carrying the light wave.

\[
\text{Time difference} \ t_2 - t_1 = \frac{\lambda}{c} \cdot \left( \frac{v^2}{c^2} \right) \cdot \left( \frac{v}{\sqrt{1-v^2}} \right)
\]

Expanding:
\[
\left[ \frac{1}{\sqrt{1-v^2}} - 1 \right] = 1 - \frac{1}{2} \left( \frac{v^2}{c^2} \right) + \ldots - 1 = -\frac{1}{2} \left( \frac{v^2}{c^2} \right)
\]

\[
\frac{1}{\sqrt{1-v^2}} = 1 + \frac{1}{2} \left( \frac{v^2}{c^2} \right) + \ldots
\]

\[
\Rightarrow \ t_2 - t_1 = \Delta t = \left( \frac{\lambda}{c} \right) \left( \frac{v^2}{c^2} \right)
\]

The phase difference is 
\( \Delta \Phi = 2\pi \left( \frac{\lambda}{c} \right) \left( \frac{v^2}{c^2} \right) \)

Since \( \frac{v^2}{c^2} = 10^{-8} \), this might be impossible, except \( \left( \frac{\lambda}{c} \right) \)

is large.
They used $L = 10$ meter and a sodium lamp with very bright yellow light at $\lambda = 600\text{nm}$.

$$\Rightarrow \frac{L}{\lambda} = \frac{10}{6} \times 10^6$$

$$\Rightarrow \Delta \phi = 2\pi \times 1.7 \times 10^{-2} \approx 0.1 \text{ radian}$$

Michelson and Morley mounted their instrument on a rotational table, so A and B could be interchanged - which should give a phase shift of $2\Delta \phi \approx 0.2$ radians. This is certainly easily observable, but no shift was ever seen.

The conclusion is that there is no "aether" medium.

Einstein postulates

1. Laws of physics - including electromagnetism - are the same in all inertial reference frames;
2. The speed of light is $c = 3.0 \times 10^8 \text{m/s}$ regardless of the relative velocities of the source and observer.

These two simple postulates have several corollaries, including:
* The velocity of light is as fast as anything can go;
* Events that are simultaneous in one inertial reference frame are not simultaneous in any other reference frame.

(3) Lorentz transformation

**Setup:**

\[
\begin{array}{ccc}
& y & \rightarrow y' \\
\downarrow & & \downarrow \\
\hline
0 & x & \rightarrow 0' \quad x' \\
& D & \rightarrow D' \\
\end{array}
\]

**Galilean**

\[
\begin{align*}
y' &= y \\
x' &= x - vt \\
t' &= t
\end{align*}
\]

**Lorentz**

\[
\begin{align*}
y' &= y \\
x' &= \gamma(x - \beta ct) \\
t' &= \gamma(c t - \beta x)
\end{align*}
\]

\[
\beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}
\]

**Transverse components are invariant**
Lorentz transformation is a generalization of the Galilean transformation. The two must be the same at low speed \((v < c)\) because the Galilean transformation works at low speed. The Lorentz transformation preserves the speed of light:

\[
ct^2 - x^2 = \text{constant}
\]

\[
\Rightarrow c^2 t'^2 - x'^2 = \sum Y^2 (c^2 t^2 - 2\beta x ct + \beta^2 x^2)
= \sum Y^2 (x^2 - 2\beta x ct + \beta^2 c^2 t^2)
= \sum Y^2 (c^2 t^2 - x^2) (1 - \beta^2) = c^2 t'^2 - x'^2
\]

The inverse transformation? Just change the sign of \(\beta\):

\[
x = Y (x' + \beta c t')
ct = Y (ct' + \beta x')
\]

(4) Simultaneity
An event means something that occurs at a specific space-time coordinate \((x, ct)\).

Suppose two events in \(O\) occur at \((0,0)\) and \((L,0)\).
Since \(t = 0\) is the same, these are simultaneous. What does observer \(O'\) see?
In \(O'\): \(\tilde{t}' = 0\)
But \(ct' = \tau (0 - \beta L) = -\beta L - it\) occurs at a time earlier than \(\tilde{t}'\), not simultaneous.

(5) Time dilation
Suppose the clock is at the origin in frame \(O\). What of frame \(O'\)?
\(ct' = \tau (ct - \beta x) \Rightarrow c dt' = \tau c dt\)
Since \(\tau > 1 \Rightarrow\) the clock, from frame \(O'\), seems to run more slowly.

This is symmetric - each observer says the other clock runs slower.
(6) Length Contraction

Consider:

\[
\begin{align*}
\quad & y \\
0 & \xrightarrow{x} Lx \\
O & \xrightarrow{t} v
\end{align*}
\]

In frame \(O'\), a detector receives signals from the two ends of \(0\)-to-\(L\) simultaneously:

\[
ct' = \gamma(c \ell - \beta x)
\]

One end is at \(x = 0\), \(t' = 0\) and the other end is at \(x = L\), \(t' = \ell\)

\[
\Rightarrow ct = \beta L
\]

\[
\Rightarrow x' = \gamma(x - \beta ct) = \gamma(L - \beta^2 \ell) = \gamma L(1 - \beta^2) = \frac{\ell}{\gamma}
\]

So the length in frame \(O'\) is shorter by a factor \((\frac{1}{\gamma})\)

(7) Addition of velocities

Consider:

\[
\begin{align*}
\quad & y \\
0 & \xrightarrow{x} x' \\
O & \xrightarrow{t} t'
\end{align*}
\]

\[
y' = y \\
u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}
\]

\[
x' = \gamma(x - \beta ct) \Rightarrow dx' = \gamma(dx - \beta c dt)
\]

\[
c \ell' = \gamma(c \ell - \beta x) \quad c dt' = \gamma(c \ell - \beta dx)
\]

\[
\Rightarrow dx' = \frac{dx - \beta c dt}{c dt' - \beta dx} = \frac{dx}{c dt - \beta dx} - \frac{\gamma u_x - \gamma v}{c(1 - (\frac{v}{c})^2)} = \frac{\beta u_x - \beta}{1 - \beta^2 x^2}
\]
Also:

\[ y' = y, \quad \text{but} \quad \frac{dy'}{d\tau} \neq \frac{dy}{dt} \quad \text{because} \quad d\tau \neq dt. \]

\[ \frac{dy'}{c\, d\tau'} = \frac{dy}{\gamma(c\, d\tau - B\, dx)} = \frac{dy}{\gamma(c - B\, U\, x)} \]