The first idea is an inertial frame of reference— one that moves at a constant velocity with respect to (w.r.t.) a fixed frame of reference. Example:

\[ \hat{x}, \hat{y}, \hat{z} \quad \rightarrow \quad \hat{x}', \hat{y}', \hat{z}' \]

Frame \( O \) \quad \quad \quad Frame \( O' \)

Viewed from frame \( O \), frame \( O' \) moves in the \( +x' \) direction at a constant speed \( v \).

(2) In our first introduction to relative motion, which we here call a Galilean coordinate transformation, all times are the same and velocities add as vectors. This leads to a coordinate transformation—assuming the coordinate systems are coincident at \( t = 0 \):

\[ x' = x - vt \]
\[ y' = y \]
\[ z' = z \]
\[ t' = t \]

Suppose that instead of \( \vec{V} = (V_x, V_y, V_z) \), a particle of mass \( m \) moves at a velocity \( \vec{u} = (u_x, u_y, u_z) \) in frame \( O \) :

\[ u_x = u \cos \theta \]
\[ u_y = u \sin \theta \]

Taking derivatives of the coordinates w.r.t. time

\[ \frac{dx'}{dt} = \frac{dx}{dt} - v = u_x' - u_x - v \]
\[ \frac{dy'}{dt} = \frac{dy}{dt} = u_y' = u_y \]
\[ u' = u \cos \theta = u \cos \theta - v \]

\[ u_y = u \sin \theta = u \sin \theta \]

\[ \tan \theta' = \frac{\sin \theta}{\cos \theta - (v/u)} \]

If I look at the inverse transformation, from frame \( O' \) to frame \( O \), I merely change the sign of \( (v) \):

\[ \tan \theta = \frac{\sin \theta}{\cos \theta + (v/u)} \]

3. In first semester freshman physics we learned about forces:

In frame \( O \): \[ F = m \frac{du}{dt} \]

If \( u' = u - v \), and \( v \) is constant, then:

\[ F' = m \frac{du'}{dt} = F \]

So as long as the force is independent of velocity, Newton's equations are the same in any inertial frame. This approach worked from the late 1500s to the late 1800s. However, in 1865 James Clerk Maxwell argued that electricity and magnetism were related. One consequence is that for an object of charge \( q \) moving at velocity \( \vec{u} \) in the presence of a magnetic field \( \vec{B} \), there is a force, called the Lorentz force:

\[ \vec{F} = q \left( \vec{u} \times \vec{B} \right) \]

This is a difficulty, because we can choose a frame \( O' \) in which the particle is at rest \( \Rightarrow \vec{F} = 0 \).

In fact, by the late 1800s people realized that classical mechanics and Maxwell's equations were incompatible.

4. Now let us return to freshman mechanics and consider a swimmer and a stream:

[Diagram of a swimmer moving in a stream]
Suppose that the swimmer's speed in still water relative to the bank is \( u > v \).

If the swimmer wants to swim straight across \( OA \), the swimmer must go upstream so that:

\[
\begin{align*}
\dot{u} &= -u \cos \theta \\
u \sin \theta &= v
\end{align*}
\]

which cancels the current \( \Rightarrow \sin \theta = \frac{v}{u} \quad \cos \theta = \left[ 1 - \left( \frac{v^2}{u^2} \right) \right]^{\frac{1}{2}} \)

To swim one round trip involves:

\[
\dot{t}_{\text{out}} = \dot{t}_{\text{back}} = \frac{L}{u \cos \theta} \quad \Rightarrow \quad \dot{t}_{\text{TOTAL}} = \left( \frac{2L}{u} \right) \cdot \frac{1}{1 - \left( \frac{v^2}{u^2} \right)^{\frac{1}{2}}}
\]

However, to swim the same distance downstream, then back upstream:

\[
\dot{t}_2 = \left( \frac{L}{u + v} \right) + \left( \frac{L}{u - v} \right) = \frac{2L}{v} \cdot \left( \frac{1}{1 - \frac{v^2}{u^2}} \right)
\]

Since \( \frac{1}{1 - \frac{v^2}{u^2}} > 1 \), \( \dot{t}_2 > \dot{t}_{\text{TOTAL}} \)

Why mention this? Because the same formulae will show up in the Michelson–Morley experiment, one of the key experiments of special relativity.

(4) Now let us consider the Doppler effect, first for sound waves. For sound waves, we must have a medium—let us use the air. Recall that the Doppler effect for a moving source is different from that for a moving observer. Let us consider both.

(a) Observer at rest, source moving:

\[
\begin{align*}
&f_0 = \text{frequency} \\
&\bullet \rightarrow \text{observer} \\
&\bullet \leftarrow \text{source at rest}
\end{align*}
\]

Let \( c = \text{speed of sound} \approx 370 \text{ m/s at STP} \)

In time \( t \), \( f_0 t \) occupy a length \( = (c - v) t \)

\[
\Rightarrow \quad \text{OBSERVER} \quad \text{wave length} \quad \lambda = \frac{c - v}{f_0} = \frac{c}{f}
\]

where \( f = \text{frequency you hear} = \frac{f_0}{1 - \frac{v}{c}} \)
As the source passes you, the observer, the sign changes to:

\[ f' = \frac{f_0}{\left(1 + \frac{v}{c}\right)} \]

(b) Observer moving, source stationary:

\[ \text{Source} \quad \longleftrightarrow \quad \text{OBSERVER} \quad \text{(c+v)} \]

\((f_0 + \lambda)\) emitted waves with wavelength \( \lambda_0 = \frac{c}{f_0}\) are travelling toward you, the observer, at any apparent speed = \((c + v)\)

\[ \Rightarrow f = \frac{(c+v)}{\lambda_0} = \left(1 + \frac{v}{c}\right) f_0 \]

Recall the expansion (for \(|x| < 1\)):

\[ \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \]

Then, to order \(\frac{v}{c}\):

source \( f = \frac{f_0}{1 + \frac{v}{c}} \) = \( f_0 \left(1 + \frac{v}{c}\right) \)

As we will soon learn, for light (speed = c) there is only relative motion. If the source and observer are approaching:

\[ f = f_0 \left(\frac{1 + \beta}{1 - \beta}\right)^{1/2} \quad \beta = \frac{v}{c} \]

For receding: \( \beta \rightarrow -\beta \) : \( f = f_0 \left(\frac{1 - \beta}{1 + \beta}\right)^{1/2} \)

If we use this for the Doppler shift of light and use the binomial expansion:

\((1+x)^n = 1 + nx + \left(n\cdot\frac{n-1}{2}\right) x^2 + \ldots \)

\[ \text{then: } f = f_0 \left(1 + \frac{\beta}{2} + \ldots\right) \left(1 + \frac{\beta}{2} + \ldots\right) \approx f_0 (1 + \beta) \]

So to first order all of the formulae are the same.
Red shift: For thousands of years the "Heavens" - the stars - were taken as constant. If the 1920 Edwin Hubble and others reported results that changed this 5,000+ year old picture. They reported a red shift of distant stars and galaxies:

\[ 0 \rightarrow \nu \]

This implies that, far from being constant, the universe is expanding. Distant objects are travelling away from us. Astronomers define the red shift as:

\[ z = \frac{f_0 - f}{f} \]

Red shifts as large as \( z = 4 \) to 5 have been observed. What velocities?

\[ z = 4 \rightarrow f_0 = 5f \rightarrow \left( \frac{1 + \beta}{1 - \beta} \right)^2 = 5 \]

\[ \Rightarrow \beta^2 = 24 \rightarrow \beta = 0.92 \Rightarrow v = 0.92c \]

Discuss dark energy, accelerating universe.