1. A person \( s' \) passes you moving at \( 0.80c \) (\( c \) = speed of light) in the \( +x' \)-direction, parallel to your \( x \)-axis.

a) He reports two events at \( x' = 0.50 \text{m} \), at times \( t'_1 = 5.00 \text{sec.} \) and \( t'_2 = 24.005 \text{sec.} \). What time difference do you observe?

Idea: Lorentz transformation (eq. 1-18 and 1-19)

\[
t_1 = \gamma (t'_1 + \frac{v x'_1}{c^2}) \quad t_2 = \gamma (t'_2 + \frac{v x'_2}{c^2})
\]

\[
\gamma = 1.67, \quad x'_1 = x'_2 \Rightarrow (t_2 - t_1) = \gamma (t'_2 - t'_1) = (1.67)(19.005) \]

\[= 31.7 \text{sec.} \]
1. (b) The origins of his and your coordinate systems coincide at $t' = 0$. He reports two events at coordinates:

\[ x_1' = 1.6 \times 10^6 \text{ m}, \quad t_1' = 8.005 \text{ s} \]
\[ x_2' = 3.40 \times 10^6 \text{ m}, \quad t_2' \]

Is there any $(t'_2)$ in his frame that would cause you to report the events as simultaneous?

Idea: Lorentz transformation (eq. 1-18 and 1-19)

\[ t_2 - t_1 = \gamma (t_2' - t_1') + \gamma^2 \left( \frac{v}{c^2} \right) (x_1' - x_2') = 0 \]

\[ \gamma = 1.07 \]
\[ (t_2' - t_1') = (t_2' - 8.005) \]
\[ (x_1' - x_2') = -1.80 \times 10^6 \]
\[ \gamma (\frac{v}{c^2}) (x_1' - x_2') = -8.02 \times 10^{-3} \]

\[ \Rightarrow (t_2' - t_1') = \frac{-8.02 \times 10^{-3}}{\gamma} = -4.8 \times 10^{-3} \]

\[ \Rightarrow t_2' = (8.005) - (0.0048) = 7.99525. \]
2. Object (A) is moving at \((-0.75c)\) in your $\hat{x}$-direction while object (B) is moving at \((+0.91c)\) in the $\hat{y}$-direction. How fast is object (A) moving in the frame of object (B)?

**Idea:** Velocity addition (eq. 1-22 and 1-23)

$$\mathbf{v}_x = \frac{(0.91 + 0.75)c}{1 + [0.91 \cdot 0.75]} = (0.987)c$$
3. \[
B + 0.91c \xrightarrow{\beta} A
\]
\[
\begin{align*}
\beta &= 0.75 \\
A &= 7.800 \times 10^{14} \text{ Hz} \\
S &= \text{your frame}
\end{align*}
\]

Light from (A) reflects from (B) and returns to (A).

a) What frequencies do you measure the light from (A) and (B) to have?

b) What frequency does (S) measure the light from (B) to have?

Idea: Doppler effect (sect. 1.5)

a) From (A):
\[
f_{\text{you}} = \left[ \frac{1 + \beta}{1 - \beta} \right]^{1/2} f_A
\]
\[
\beta = \frac{v}{c} = \frac{0.75}{1} = 0.75
\]
\[
f_{\text{you}} = 7.408 \times 10^{14} \text{ Hz}.
\]

From (B): (B) see the source = (A) approaching at speed
\[
\frac{0.91c + 0.75c}{1 + \left[ \frac{0.91c}{0.75c} \right]} = 0.987c = \beta
\]

So it detects a frequency = \(3.462 \times 10^{15}\) Hz. This frequency reflects from (B) and it what goes to you, with the source = (B) moving at \(v = 0.91c\) \(\Rightarrow \beta = 0.91\)

\[
f_{\text{you}} = \left[ \frac{1 + \beta}{1 - \beta} \right]^{1/2} f_{\text{REFLECT}} = 1.595 \times 10^{16} \text{ Hz}.
\]

b) From (B), \(f_{\text{source}} = 3.462 \times 10^{15}\) Hz. This is what reaches (A), with a source moving at \(\beta = 0.987\)

\[
f_A = \left[ \frac{1 + \beta}{1 - \beta} \right]^{1/2} f_S = 4.280 \times 10^{16} \text{ Hz}.
\]
4. Text problem 2-39. I would select 1 or 2 of the 4 parts.

a) Idea: Length contraction

\[ \gamma = \frac{50 \times 10^9 \text{eV}}{0.511 \times 10^6 \text{eV}} = 9.8 \times 10^4 \]

=> Bundle length = (1cm) \((9.8 \times 10^4) = 980 \text{ m} \)

b) Idea: Lorentz transformation

Velocity addition

\[ \gamma = 9.8 \times 10^4 \quad 9.8 \times 10^4 \quad e^+ \]

\[ v = 0.99999c \quad v = 0.99999c \]

Electron energy = \(\delta (mc^2) = (2.0 \times 10^5)(5.11 \times 10^3) = 1.02 \times 10^{11} \text{ eV} \)

Momentum: \( E_{\text{total}} = \gamma mc^2 = \left[ \sqrt{(pc)^2 + (mc^2)^2} \right]_{\frac{1}{2}} \)

\[ \Rightarrow \gamma (mc^2)^2 = (pc)^2 + (mc^2)^2 = \frac{1}{2}(pc)^2 = \frac{1}{2} (\gamma^2 - 1)(mc^2)^2 \]

\[ \Rightarrow pc = \sqrt{(\gamma^2 - 1)} (mc^2) = 1.02 \times 10^{11} \text{ eV} \]

\[ \Rightarrow p = \frac{1.02 \times 10^{11} \text{ eV}}{c} \]
5. An electron (rest energy 0.511 MeV) increases its speed to the point that its total energy is 2.607 MeV.

a) What is the $\gamma$-factor?

b) What is the kinetic energy?

c) What is the linear momentum?

Idea: Relativistic energy.

\[ \gamma = \frac{2.607}{0.511} = 5.102 \]

\[ KE = (\gamma - 1)(mc^2) = (4.102)(0.511\text{ MeV}) = 2.096\text{ MeV} \]

\[ p = \gamma mc = \sqrt{\gamma^2 - 1} (mc^2) = 2.557\text{ MeV} \]

\[ p = \frac{2.557\text{ MeV}}{c} \]
6. A subatomic particle \( X \) has a rest energy in your laboratory frame of \( 279 \text{ MeV} \) and is at rest. It is reported that \( X \) decays into two particles, \( Y \) (rest energy \( 102 \text{ MeV} \)) and \( Z \) (rest energy \( 150 \text{ MeV} \)). Tell me either why this is possible or not possible.

Two conditions:
- Energy conserved
- Linear momentum conserved.

Energy:
\[
279 \text{ MeV} = \left[ (\gamma_Y)(102 \text{ MeV}) \right] + \left[ (\gamma_Z)(150 \text{ MeV}) \right]
\] (1)

Linear momentum:
\[
P_Y = \sqrt{(\gamma_Y^2 - 1)} \cdot (102 \text{ MeV}) = P_Z \]
\[
\Rightarrow \sqrt{(\gamma_Y^2 - 1)} = \frac{P_Z}{c} = \frac{\sqrt{(\gamma_Z^2 - 1)} \cdot (150 \text{ MeV})}{c}
\] (2)

From Energy:
\[
\gamma_Y = \sqrt{\left( \frac{279}{102} \right)} - \left[ \left( \frac{150}{102} \right) \right] \] (3)

Substitute (5) into (4):

\[
\gamma_Z = 0.537
\]
\[
\gamma_Y = 1.945
\]

So there is a solution \( \Rightarrow \boxed{ \text{possible} } \)