Physics 202 Exam 2 (Hokin)
Wednesday, April 10, 2013

Name SOLUTIONS

Hokin

Problem 1 (power line)
Problem 2 (tokamak)
Problem 3 (wire and rectangular loop)
Problem 4 (tank circuit)
Problem 5 (series RLC circuit)

TOTAL / 100
1. A horizontal power line of length 58.0 m carries a current of 2.20 kA in the northward direction as shown. The earth's magnetic field at this location is \( 0.5 \times 10^{-5} \) T. The field at this location is directed downward and toward the north at an angle of 65.0° below the power line.

(a, 5 pts) Show the direction of the magnetic force on the power line using standard vector notation.

\[
\text{see figure} 
\]

\[ F = l \hat{I} \times \vec{B} = l IB \sin \phi \]
\[ = (58.0 \text{ m}) (2.20 \text{ kA}) (0.5 \times 10^{-5} \text{ T}) \sin 65^\circ \]
\[ F = 5.48 \text{ N} \]

3 pts: \( F = l \hat{I} \times \vec{B} \) or \( I S d \hat{z} \times \vec{B} \)
6 pts: \( F = I L B \sin \phi \) or determinant

(b, 10 pts) Determine the magnitude \( F \) of the magnetic force on the power line.

9 pts: Correct numerical answer

(c, 5 pts) Given that the wire is 12.0 m above the ground, calculate the magnetic field strength \( B_{\text{wire}} \) due to the wire at ground level. How does \( B_{\text{wire}} \) compare with earth's magnetic field in magnitude and direction? (Note: drawing is not to scale – the wire is almost five times longer than it is high above the ground.)

1 pt: \( B_{\text{wire}} \)
2 pts: correct number w/units

\[ B_{\text{wire}} = \frac{\mu_0 I}{2 \pi h} = \frac{4\pi \times 10^{-7} \text{ Tm} \times 2.20 \text{ kA}}{2 \pi (12.0 \text{ m})} = 3.67 \times 10^{-5} \text{ T} \]

1 pt: \( B_{\text{wire}} \)
1 pt: Length wire formula

1 pt: \( 2 \pi (12.0 \text{ m}) \)

1 pt: Direction comparison (specify dir)

Fields are comparable in magnitude, but
\[ B_{\text{wire}} \neq B_{\text{wire}} \text{ at ground} \]
2. The magnetic coils of a tokamak fusion research experiment are in the shape of a toroid having an inner radius \( R_{\text{min}} \) and an outer radius \( R_{\text{max}} \). The toroid has \( N \) turns of thick wire, each of which carry a current \( I \) in the direction shown.

(a, 5 pts) Indicate the direction of the magnetic field inside the toroid.

Correct or not (Only 3 for extra incorrect information included)

(b, 15 pts) Determine the magnetic field \( B(R) \) as a function of radius within the toroid, \( (R_{\text{min}} < R < R_{\text{max}}) \), using Ampère's Law. Draw your Ampère's Law loop on the diagram.

- 5 points for correct Ampère loop
- 2 points Ampère Law
- 2 points \( \mu_0NI \) for \( I_{\text{in}} \)
- 2 points for \( \int B \cdot ds \) integral setup
- 2 points for correct evaluation of \( \int B \cdot ds \) integral
- 2 points for correct answer

\[ \int B \cdot ds = B(R) 2\pi R = \mu_0I_{\text{in}} = \mu_0NI \]

\[ B(R) = \frac{\mu_0NI}{2\pi R} \]
3. A rectangular coil of dimensions $l$ and $w$ and $N$ turns moves with a constant velocity $v$ away from a long wire that carries a current $I_{wire}$ in the plane of the loop, as shown. The resistance per length of the coil wire is $\rho$.

(a, 5 pts) Determine the magnetic flux $\Phi(r)$ inside the coil, starting from basic equations on the formula sheet (you may already know the answer, but you must derive it here).

$$\Phi(r) = \int_0^l \frac{\mu_0 I_{wire}}{2\pi r} l dr = \frac{\mu_0 I_{wire} l}{2\pi} \int_0^{r+w} \frac{dr}{r}$$

$$= \frac{\mu_0 I_{wire} l}{2\pi} \ln \left(\frac{r+w}{r}\right)$$

- Correct $\Phi = \frac{\mu_0 I_{wire} l}{2\pi r}$ (1)
- Correct $\Phi$ from $B$ (5), Calculation errors (-1)

(b, 10 pts) Determine the total EMF $\mathcal{E}(r)$ induced between the start and end of the coil's wire using your solution from (a). Indicate the direction of $\mathcal{E}$ on the diagram.

$$\mathcal{E} = -N \frac{d\Phi}{dt} = -N \frac{\mu_0 I_{wire} l}{2\pi} \frac{1}{1 + \frac{w}{r^2}} \frac{dr}{dt}$$

$$\mathcal{E} = +N \frac{\mu_0 I_{wire} l}{2\pi} \frac{w}{r+w}$$

- Direction of emf (1)
- Wrong calculations (-1 -3)
- $\mathcal{E} = -N \frac{d\Phi}{dt}$ (1)
- Wrong derivatives (-1 -3)

(c, 5 pts) Determine the current $I_{loop}(r)$ in the coil's wire using your solutions from (a) and (b). Assume that the inductance of the loop can be ignored (i.e., that its impedance is purely resistive). Please use the wire and loop subscripts to keep the two currents separate!

$$R = N \rho (2l + 2w) = 2N \rho (l+w)$$

$$I_{loop} = \frac{\mathcal{E}}{R} = \frac{\mu_0 I_{wire} l}{4\pi \rho (l+w)} \frac{1}{r+w} \frac{1}{2N \rho (l+w)}$$

- Wrong emf, Wrong procedure (1)
- Correct procedure (2)
- Wrong resistance $\rho$ (1)
- Missing $N$ (-1)
- etc (-2)
4. In the circuit shown, the battery EMF is 50.0 V, the resistance is 250 Ω and the capacitance is 0.500 μF.

(a, 5 pts) The switch S is closed for a long time so that the current through the inductor is constant. Determine the rates of power loss in the resistor, \( P_R \), inductor \( P_L \) and capacitor \( P_C \).

\[
I = \frac{E}{R}, \quad P_R = I^2R = \frac{E^2}{R} \quad (3)
\]

\[
P_L \quad (1)
\]

\[
P_C = 0 \quad (always) \quad (1)
\]

(b, 15 pts) After the switch is then opened, the potential difference across the capacitor reaches a maximum value of 150 V. Determine the value of the inductance.

**Energy Conservation**

\[
V_{tot} = \frac{1}{2} LI^2 \text{ at start} = \frac{1}{2} L \left( \frac{E}{R} \right)^2 \quad (3)
\]

Max voltage on cap \( = V_{tot} = \frac{1}{2} CV_{max}^2 \quad (3) \)

\[
\Rightarrow \quad \frac{1}{2} L \left( \frac{E}{R} \right)^2 = \frac{1}{2} CV_{max}^2 \quad (5)
\]

\[
L = \left( \frac{R}{E} \right)^2 CV_{max}^2 = 281 \text{ mH} \quad (2)
\]

\[
L = \frac{CV_{max}}{I^2} \quad (2)
\]
5. Consider a series \( RLC \) circuit having the parameters \( R = 200 \, \Omega \), \( L = 663 \, \text{mH} \) and \( C = 10.6 \, \mu\text{F} \). The applied voltage has amplitude \( V_{\text{max}} = 50.0 \, \text{V} \) and frequency \( f = 60.0 \, \text{Hz} \).

(a, 5 pts) Determine the amplitude of the current \( I_{\text{max}} \) and its phase \( \phi \) relative to the applied voltage.

\[
R = 200 \, \Omega \\
X_L = \omega L = 2\pi f L = 250 \, \Omega \\
X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 250 \, \Omega \\
\Rightarrow \mathbb{Z} = R \Rightarrow I_{\text{max}} = \frac{V_{\text{max}}}{R} = 250 \, \text{mA} \quad \phi = 0
\]

(b, 10 pts)
Determine the maximum voltage \( V_R \) across the resistor and its phase \( \phi_R \) relative to the current.
Determine the maximum voltage \( V_C \) across the capacitor and its phase \( \phi_C \) relative to the current.
Determine the maximum voltage \( V_L \) across the inductor and its phase \( \phi_L \) relative to the current.

\[
V_R = I_{\text{max}} R = V_{\text{max}} = 50.0 \, \text{V} \quad \phi_R = 0
\]

\[
V_C = I_{\text{max}} X_C = 62.5 \, \text{V} \quad \phi_C = 90^\circ \quad \text{after current}
\]

\[
V_L = I_{\text{max}} X_L = 62.5 \, \text{V} \quad \phi_L = 90^\circ \quad \text{before current}
\]

(c, 5 pts) Draw a phasor diagram showing \( V_R \), \( V_C \), \( V_L \), \( V_{\text{max}} \) and \( \phi \).

\[50.0 \, \text{V} = I_{\text{max}} R \]

\[V_C = V_{\text{max}} \]

\[I_{\text{max}} X_L = 62.5 \, \text{V} \]

\[I_{\text{max}} X_C = 62.5 \, \text{V} \]