CH 37 - WAVE OPTICS - INTERFERENCE

Now look at wave properties in interaction with matter - matters when wavelength vs size

\begin{align*}
\text{light} & \bigg| \text{slit widths} \not\text{ not bent} \\
\text{light} & \bigg| \text{slit widths bent}
\end{align*}

\[ \text{slit huge, say } 10^5 \lambda \]
Ray approximation good \[ \lambda \gg s \]

\[ \text{narrow slit, say } 2 \lambda \]
Ray approximation bad \[ \lambda = s \]
\[ \Rightarrow \text{ wave optics needed} \]

Demo: Double Slit
\[ \text{Pisces Patterns} \]

\[ \text{"Fringes" = bands of light/dark} \]

Sources in phase (double slit)

\[ S_1 \quad S_2 \]

\[ \text{extra path length traveled by } 2 = \delta = r_2 - r_1 \approx d \sin \Theta \]

Two-Slit/Source Interference

\[ I(\Theta) \quad L \sin \Theta \]

\[ \text{(a) coherent light} \]
\[ \text{set phase relation} \]
\[ \text{(b) monochromatic} \]
\[ \text{only one } \lambda \]

Constructive Interference
when \[ s = m \lambda \quad (m = 0, \pm 1, \pm 2, \ldots) \]
\[ \text{"far-field" approximation} \]
\[ \Rightarrow \quad d \sin \Theta = m \lambda \quad \text{two-source interference} \]
\[ \text{(or any number of sources, } d \text{ between each)} \]
Example: Double antenna Radio Station

Radio Moscow
\[ f = 11,980 \text{ kHz} \]
\[ \lambda = 25.842 \text{ m} \]

"25-meter bond" strong signal \( A_1, A_2 \) in phase

Strong when \( \sin \theta = n \times \frac{2\pi}{m} \)
\[ \sin \theta = 0, \pm 1 \]

How can direction of strong signal be adjusted??

\[ \Rightarrow \text{Ans: adjust } \Delta \phi \text{ between sources!} \]

\[ \Rightarrow \text{Aim signal at targeted audience, e.g. NA} \]

Can also view distance of fringes from center line on screen:
\[ y = L \sin \theta = \frac{m \lambda}{d} \]
(sources in phase)

(small angles)

\[ y = L \sin \theta \]

\[ \Delta \phi \text{ between sources} \]

\[ \Rightarrow \text{Aim signal at targeted audience, e.g. NA} \]

\[ \text{INTENSITY DISTRIBUTION} \]

\[ E_1 = E_0 \sin (\kappa r_1 - wt) = E_0 \sin \phi_1 \]
\[ E_2 = E_0 \sin (\kappa r_2 - wt) = E_0 \sin \phi_2 \]

\[ \phi_1 = \kappa r_1 - wt = \text{phase of wave 1 at observer} \]

\[ \phi_2 = \kappa r_2 - wt = \text{phase of wave 2 at observer} \]

\[ \Rightarrow E = E_1 + E_2 = E_0 \left( \sin \phi_1 + \sin \phi_2 \right) \]

\[ \phi_1 + \phi_2 = \frac{\kappa (r_1 + r_2)}{2} - 2wt \]
\[ \frac{\phi_1 + \phi_2}{2} = \frac{\kappa (r_1 + r_2 + s)}{2} - 2wt = \frac{\kappa (r_1 + r + s)}{2} \]
\[
\frac{\phi_1 - \phi_2}{2} = \frac{kr_1 - wt - kr_2 + wt}{2} = \frac{k}{2} (r_1 - r_2) = -\frac{kS}{2}
\]

\[\Rightarrow E(\theta, t) = E_0 \, 2 \sin \left( kr_1 - wt + \frac{kS}{2} \right) \cos \left( \frac{krS}{2} \right)\]

Intensity \( I \propto E^2 = 4E_0^2 \sin^2 \left( kr_1 - wt + \frac{kS}{2} \right) \cos^2 \left( \frac{krS}{2} \right) \)

oscillates in time constant in time

\[
\langle I \rangle \propto 4E_0^2 \frac{1}{2} \cos^2 \left( \frac{krS}{2} \right)
\]

\(\text{(drop angles)} \quad I = I_{\text{max}} \cos^2 \left( \frac{krS}{2} \right)\)

\[S = d \sin \theta\]

Angular Intensity Distribution

Constructive when \( \frac{krS \sin \theta}{2} = m \pi \quad (\cos^2 (m\pi) = 1) \)

\[\Rightarrow d \sin \theta = \frac{2\pi m}{k} = m \lambda \]

Wave Reflection

A wave gets \(180^\circ = \pi \) phase shift when reflects off of higher-\(n\) surface (so, not internal reflection)