Image Formation by Refraction

point on axis: $p > 0$
real object in front

Snell's Law:

$$n_i \sin \theta_i = n_r \sin \theta_r$$

+ trig on $x, y, \theta$

$$R = \frac{n_i + n_r}{n_i - n_r} \frac{b}{\theta}$$

+ small $x \Rightarrow$ small $\theta_i, \theta_r$

Single interface

use caution:

If flat $\Rightarrow R = \infty \Rightarrow \frac{n_i}{p} + \frac{n_r}{b} = 0$

If $p > 0$ (real) object in front
If $p < 0$ (virtual) object in back
If $q > 0$ (real) image in back
If $q < 0$ (virtual) image in front
If $q > 0$ image erect
If $q < 0$ image inverted

$\theta_i, \theta_r$: angles

$R$: radius of curvature

$I$: image

$\theta_i, \theta_r$: angles

$R$: radius of curvature

$I$: image

Examples:
- Water surface: "fish" looks closer
Image Formation by Thin Lenses

Assume \( \eta_{\text{air}} = 1 \Rightarrow n = n_{\text{of glass}} \)

Single interface:

\[
\frac{1}{p} + \frac{n}{q} = \frac{n-1}{R}
\]

\[p_1 \quad q_1 \quad R_1 \]

(THICK)

\[R_2 \quad \eta = n = 1 \quad \eta = n = 1 \]

Final real inverted image

\[p_2 \quad q_2 \quad R_2 \]

\[
\frac{1}{p} + \frac{n-1}{q} = 1 - h
\]

Virtual object for second surface:

\[
\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}
\]

\[
\frac{1}{p_2} + \frac{n}{q_2} = \frac{1-h}{R_2}
\]

Small \( t \) \Rightarrow measure all lengths from center of lens:

\[p_2 = -q_1 \quad (q_1 < 0, \text{ virtual in front})\]

\[q_2 = "q" \quad \text{for final image}\]

\[p_1 = "p" \quad \text{for real object}\]

\[
\frac{1}{p} + \frac{1}{q} (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

If \( p \rightarrow \infty \) then \( q \rightarrow f \)

Place where parallel rays cross axes

"Lens Maker's Eqn"

\[
\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

Thin Lens Equation
Ray Diagrams - Thin Lenses

Some basic rays as mirrors:
(A) parallel ray goes through focus (front/back)
(A') - really, for converging lens
- virtually, for diverging lens
(c) ray through center of lens does not bend

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}
\]

\[
M = \frac{h'}{h} = -\frac{q}{p}
\]

Convexing Lens \( f > 0 \)

Diverging Lens \( f < 0 \)

Real \( h' > 0 \)
Inverted \( h' < 0 \)
Virtual \( h' > 0 \)
Over \( h' < 0 \)
Lens Combinations: image of first lens is object of second lens

1. Virtual erect object for lens 2

2. Final image is virtual, erect, happens to appear to be at lens 1 (coincidence)

Math:
\[ \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \]
\[ \frac{1}{g_1} = \frac{1}{f_1} + \frac{1}{p} \]

\[ f_1 = 2.9 \text{ in} \quad \Rightarrow \quad \frac{1}{f_1} = \frac{1}{2.9} = -\frac{1}{2.9} \]
\[ p_1 = 2.2 \text{ in} \quad \Rightarrow \quad \frac{1}{p_1} = \frac{1}{2.2} \]
\[ s = 1.9 \text{ in} \quad \Rightarrow \quad q_1 = -9.1 \text{ in} \text{ from lens 1 (in front)} \]

\[ M_1 = -\frac{q_1}{p_1} = 4.1 \quad \text{\(\checkmark\) erect (virtual)} \]
Now Second lens (divergent) \( f_2 = -2.2 \text{ in} \)

\[
\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}
\]

\[
p_2 = -8.1 + 5 = -3.1 \text{ in}
\]

\[
q_2 = 9.1 \text{ in} + 1.9 \text{ in} = 11.0 \text{ in}
\]

\[\frac{1}{q_2} = \frac{1}{f_2} \Rightarrow \frac{q_2}{f_2} = 1 \]

\[
\frac{1}{q_2} = \frac{1}{f_2} \Rightarrow \frac{q_2}{f_2} = 1
\]

\[
\frac{q_2}{f_2} = \frac{-2.2 \text{ in}}{11.0 \text{ in}} = 0.18 \text{ in}
\]

\[
\frac{1}{q_2} = \frac{1}{f_2} \Rightarrow \frac{q_2}{f_2} = 1
\]

\[
\Rightarrow q_2 = -1.83 \text{ in} \quad \text{from lens 2} \quad \text{line up with page 4 holes}
\]

\[
M_2 = -\frac{q_2}{p_2} = 0.17 \quad \text{evert (virtual)}
\]

\[
\Rightarrow M = M_1M_2 = 0.70
\]

(Pretty dumb lens combination, but instructive.)