Motion $EMF = E$ from "cutting field lines"

Books gets this wrong

Fundamental View

Rod "cuts field lines"

From point of view in $\overrightarrow{B}$ frame of reference: $F_B = q \overrightarrow{v} \times \overrightarrow{B}$ force pushes $\Theta \uparrow \Theta \downarrow$, what it

From rod's frame of reference: $\overrightarrow{v} = 0$, $\overrightarrow{v} \times \overrightarrow{B} = 0$! But field lines sweeping by at $\overrightarrow{v}$ induce electric field $E_{ind} = \overrightarrow{v} \times \overrightarrow{B}$ ($\overrightarrow{v} = \overrightarrow{B}$ line motion)

Explains magnetic force $\Rightarrow E = -\overrightarrow{v} \overrightarrow{B}$ (don't need conductor!)

Another version:

$E$ direction drives $I$ to oppose change in $\Phi$

Energy: $I = \frac{E}{R}$; $P_R = \frac{E^2}{R} = \frac{v^2 L^2 B^2}{R} = \frac{P}{R}$ "out"

Input $P_i = F_B v = (ILB) v = \frac{E}{R} (ILB) = \frac{v^2 L^2 B^2}{R}$ "in"
What if $E_{\text{ind}}$ isn't constant along path?

Recall $\mathcal{E} = \int E \cdot ds$ (potential-field relation)

- General form: $\int E \cdot ds = -\Phi_0$ (no big deal)

**Generators and Motors**

**Generator:** Convert mechanical power to electrical power via induction and $\mathbf{I} \times \mathbf{B}$ force

**Motor:** Convert electrical power to mechanical power via induction and $\mathbf{I} \times \mathbf{B}$ force

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**Rail Demo #1**

**Linear Motor**

Close switch $\Rightarrow \mathbf{I} \times \mathbf{B}$ force $\rightarrow$ rail driven to right

What is $V_{\text{max}}$? (If zero friction)

When back EMF of motion cancels applied $V$

\[ V + E = 0 \quad \Rightarrow \quad -E = VBL = V \]

\[ V_{\text{max}} = \frac{V}{BL} \text{ independent of mass!} \]

**LM - Train:**

\[
\begin{align*}
V &= 120 \text{ V} \\
B &= 1.0 \text{ T} \\
L &= 1.5 \text{ m (track gauge)}
\end{align*}
\]

\[ V_{\text{max}} = 80 \text{ m/s (w/ drag)} \text{ longer to reach } V_{\text{max}} \]

\[ = 288 \text{ km/h} \]

\[ = 177 \text{ mi/h} \]
Rail Demo #2

Apply force to bar \( \rightarrow \) speed \( v \)

\[ E = vBL \text{ measured by voltmeter} \]

If no current \( \Rightarrow \) no \( I \times B \) force \( \Rightarrow \) no opposing force

Linear Generator:

Drive current \( I \)

Load \( R \)

\( e.g., \) \( V \) \( \) \( E \)

\[ I = \frac{E}{R} \quad F_B = BLI \quad E = vBL \]

\( v \) is constant when \( F_B = F_{app} \)

\[ BL = \frac{F_{app}}{I_{max}} = \frac{E_{max}}{BL} \]

\[ \frac{BE}{R} L = \frac{F_{app} R}{BL} \]

\[ B \frac{vBL}{R} L = F_{app} \]

\( \Rightarrow \) \( v_{max} = \frac{F_{app} R}{R^2 L^2} \) (if no friction)

Output power \( P_{max} = I_{max}^2 R \)

\[ = \frac{F_{app}^2}{B^2 L^2} R \]

AC generator, rotating loop in DC \( B \) field

\[ \Phi = BA \cos \omega t \]

\[ \dot{\Phi} = -BA \omega \sin \omega t \]

\[ E = -N \dot{\Phi} = NBA \omega \sin \omega t \]
Eddy Currents - induced currents in conductors

Classic: eddy current pendulum

Break eddy currents with slots

\[ E \Rightarrow I \Rightarrow I \times B \]

force opposes motion

no slots

slots, I can't flow well.