Electric Field

Field is a handy way to represent force independent of quantity that force acts on.

We've already seen gravity field \( \vec{g} \): near Earth \( \vec{g} = -g \hat{y} \).

force \( \vec{F} = m \vec{g} \) field stuff field acts on around a planet \( \vec{F} = -G \frac{Mm}{r^2} \hat{r} \).

Electric field is same idea \( \vec{F} = q \vec{E} \) from other charges e.g. point charge, Coulomb law

\( \vec{F}_e = k_e \frac{qQ}{r^2} \hat{r} \Rightarrow \vec{E} = \frac{k_e q}{r^2} \hat{r} \).

You can add \( \vec{E} \) from a number of charges, to get total \( \vec{E} \).

Example 23.5

\( \vec{E}_2 + \vec{E} = \vec{E}_1 + \vec{E}_2 \).
Continuous charge distribution

Add up the electric field contribution $d\mathbf{E}$ from each charge element $dq$ due to Coulomb's law.

\[
d\mathbf{E} = \frac{kq dq}{r^2}
\]

\[
(d\mathbf{E} = k \frac{dq}{r^2})
\]

\[
\Rightarrow \mathbf{E} = \int \frac{kq dq}{r^2}
\]

all charge

Volume charge: $dq = \rho \, dV$ \quad $\rho =$ charge density $\frac{C}{m^3}$

Surface charge: $dq = \sigma \, dA$ \quad $\sigma =$ charge density $\frac{C}{m^2}$

Line charge: $dq = \lambda \, dx$ \quad $\lambda =$ linear charge density $\frac{C}{m}$

ex. Charged rod (uniform) with $Q$ over $L$

\[
dq = \lambda \, dx \quad \lambda = \frac{Q}{L}
\]

distance $= (x-x') \Rightarrow d\mathbf{E} = \frac{k dq}{(x-x')^2} = \frac{k dq}{(x-x')^2} \quad$ integrate over rod
\[ E = \int_{a}^{b} dE = \int_{-L}^{L} k e \lambda \frac{dx'}{(x-x')^2} \]

\[ = ke \lambda \frac{1}{(x-x')} \bigg|_{-L}^{0} = ke \lambda \left( \frac{1}{x} - \frac{1}{x+L} \right) \]

\[ E(x) = ke \lambda \frac{L}{x(x+L)} = \frac{keQ}{x(x+L)} \quad (\lambda = \frac{Q}{L}) \]

**Electric Field Lines** - connect \( \vec{E} \) vectors moving away from source charge

Note that lines are closer together where \( \vec{E} \) is stronger

- geometric interpretation (explanation) of \( 1/r^2 \) force law
- \( \vec{E} \) field lines

Spherical Surface: \( 4\pi r^2 \propto 1/r^2 \)

\[ \Rightarrow \text{lines surface} \propto \frac{1}{r^2} \quad \text{(Same for gravity)} \]
Since $E = \text{charge, number of lines}$ from a source & charge = double the charge double the number of lines you draw.

**DIPOLe**

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Total charge = 0 $\Rightarrow$ all lines from $+Q$ terminate on $-Q$ (including wrap at $\infty$).

**DIPOLe + MONOPOLe**

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Note: from far away, it looks like $+Q$ (not charge).
Motion of charge in electric field

Review of 201
Combine \( \vec{F} = \vec{ma} \) with \( \vec{F} = q \vec{E} \)
\[ \Rightarrow \quad \vec{ma} = q \vec{E} \]
\[ \Rightarrow \quad \vec{a} = \frac{q}{m} \vec{E} \]
\( \vec{E} \) given by other charges (or, later, induction)

Ex. Old TV tube, \( E = \frac{10^4 \text{V}}{1 \text{cm}} = 10^6 \text{V/m} = 10^6 \text{N/C} \)
\[ \Rightarrow \quad a = \frac{q}{m} E = 1.8 \times 10^{-15} \text{m/s}^2 \]
Huge but short lived

Electron picks up 10 keV energy = 1.6 \times 10^{-15} J

Speed?
\[ \frac{1}{2} m v^2 = Fx = eEx = max \]
\[ v^2 = 2ax = 2 \times 1.8 \times 10^{-17} \text{m} \times 10^2 \text{m/s}^2 \]
\[ v^2 = 3.6 \times 10^{-15} \text{m}^2/\text{s}^2 \]
\[ v = 6.0 \times 10^{-7} \text{m/s} < 3 \times 10^8 \text{m/s} \]
(very fast, but not relativistic)