1) Find the energies and wavefunctions for a massless particle moving in the x-direction with energy $E$. Classify your solutions in terms of their helicity and associate them with particles or antiparticles.

2) Rewrite the Dirac equation for arbitrary mass in the basis of eigenstates from 1).

3) For negative helicity particles, use first-order perturbation theory to find the energy shift.

4) Given a solution $\psi(x)$ to the Dirac equation in one reference frame, find $\psi'(x') = S\psi(x)$ ($S$ is a 4X4 matrix) in a reference frame moving with velocity $v\hat{x}$, where $S = a + b\gamma^0\gamma^1$, where

\[ a = \frac{1}{\sqrt{2}}(\gamma + 1), \quad b = -\frac{1}{\sqrt{2}}(\gamma - 1). \]

To do this, we write the Dirac equation in the moving frame:

\[ (i\gamma^\nu \partial'_\nu - m)\psi' = 0 \]

and transform back to the lab frame:

\[ (i\gamma^\mu \Lambda^\nu_\mu \partial'_\nu - m)S^{-1}\psi' = 0 \]

Multiply on the left by $S^{-1}$ and equate to get an equation like

\[ SB^\mu = \gamma^\mu S \]

and verify it for the four values of $\mu$.

5) Show, using $S$ from the previous problem, that $\bar{\psi}\psi$ is a relativistic invariant.