Phys 449 HW 4 Due Feb 17

1) A three-level atomic system ($E_a < E_b < E_c$) interacts with oscillating fields via $V = \varepsilon_a e^{-i\omega_a t} |c\rangle\langle a| + \varepsilon_b e^{-i\omega_b t} |c\rangle\langle b| + \text{h.c.}$, with $\hbar(\omega_a - \omega_b) = E_b - E_a$.
   a. Find an effective time-independent Hamiltonian using an appropriate transformation.
   b. Find an eigenstate of the system that has zero component of $|b\rangle$ ("dark" state).
   c. Suppose $\varepsilon_a, \varepsilon_b$ slowly vary in time. At $t = 0$, $\varepsilon_a = 0, \varepsilon_b > 0$, assume the atom is in the dark state. Later, at $t = t_1$, $\varepsilon_b = 0, \varepsilon_a > 0$. How large must $t_1$ be in order that the atom remain in the dark state? Hint: think about Fourier analysis and the Bohr criterion.

2) BD 16.1
3) BD 16.2
4) BD 16.3 The chemical potential can be thought of as a normalizing factor that guarantees $\sum_{nG} n_G = N$.
5) Construct the appropriately symmetrized wavefunction for the ground state of 3 spin-$1/2$ particles in a harmonic oscillator potential.