last time: optics w/ mirrors + planar refraction
today: example w/ mirror + lenses

example

Consider an object 5 cm tall placed 55 cm from a converging mirror with focal length 20 cm.

a) Where is the image produced?

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = \left( \frac{1}{f} - \frac{1}{d_0} \right)^{-1} \]

\[ = \left( \frac{1}{20} - \frac{1}{55} \right)^{-1} \text{cm} \approx 3/\text{cm} \]

b) What is the magnification?

\[ m = \frac{h_i}{h_0} = -\frac{d_i}{d_0} \approx -\frac{3\text{cm}}{55\text{cm}} \approx -0.6 \]

since \( h_i < 0 \) \( \leftrightarrow \) inverted

\[ \text{c) What is the size of the image?} \quad h_i = m h_0 \approx (-0.6)(5\text{cm}) = -3\text{cm} \]
Parabolic mirrors

"spherical aberration"

If parabola $= y = ax^2$, the focal point is at $(0, \frac{1}{4a})$. 

$y = \frac{1}{2} (1 - \sqrt{1 - 4x^2})$

$y = x^2$
Layers of indices of refraction can produce interesting phenomena. 

Example:

\[ n_{\text{hot}} \sin \theta_{\text{u}} = n_{\text{cold}} \sin \theta_{\text{d}} \]

If \( \frac{n_{\text{hot}} \sin \theta_{\text{u}}}{n_{\text{d}}} > 1 \), there is no solution \( \Rightarrow \) reflection

(Total internal reflection in fiber optic cables also rely on \( \frac{n_{\text{i}} \sin \theta_{1}}{n_{2}} > 1 \))

Next, shape of refraction boundary \( \rightarrow \) lenses
Thin Lenses: Any lens whose thickness is small compared to any other length scale one is interested in (e.g., object distance) as well as the radii of curvature of lens surfaces.

Lens-Maker's formula:
- \( \frac{1}{f} = (n_{\text{glass}} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \)
- \( \frac{1}{f} = \left( \frac{n_{\text{medium}} - 1}{R_1} \right) \)
- \( \frac{1}{f} = \left( \frac{n_{\text{medium}} - 1}{R_2} \right) \)
- \( \frac{1}{f} = \left( \frac{n_{\text{medium}} - 1}{R_3} \right) \)

Thin-lens equation:
- \( \frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \)
sign conventions assuming light goes from left to right
- \( d_o > 0 \) for left of lens and \( d_o < 0 \) for right of the lens
- \( d_i > 0 \) for real images; \( d_i < 0 \) for virtual images
- If the image is upright, \( h_i > 0 \). If the image is inverted, \( h_i < 0 \).

**Example**

An object w/ height \( h_o = 5 \) cm is placed at a distance \( d_o = 16 \) cm from a thin converging lens with focal length \( f = 4 \) cm.

a) What is the image distance?

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

\[
d_i = \left( \frac{1}{f} - \frac{1}{d_o} \right)^{-1} = \left( \frac{1}{4} - \frac{1}{16} \right)^{-1} \text{ cm} = \frac{4}{3} \text{ cm}
\]

b) What is the magnification?

\[
m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = \left( \frac{1}{16} \right)^{-1} \times \frac{4}{3} = \frac{-1}{3}
\]

c) What is the image height?

\[
h_i = mh_o = \frac{-5}{3} \text{ cm}
\]
Examples of 2-lens ray tracing

Compound microscope

Intermediate image lies beyond the 2nd lens