Topics: Faraday's law of electromagnetic induction (Parcell Ch 7)

Up to this point → we've focused on static electromagnetic fields
now we begin the study of electrodynamics

which began with the discovery of Faraday of electromagnetic induction in 1830s

his query: given that presence of charges leads to rearrangement of charges on nearby conductors, does an electric current induce a current in another conductor?

Experimentally, he found:

- Such currents were induced, but only by a changing current
- Induced current acts to oppose the change of the original current

induced current → induced emf in circuit

(emf: work done/charge \( \frac{1}{q} \oint \mathbf{E} \cdot d\mathbf{s} \) around the closed loop of circuit)
Consider a loop moving through a \( \vec{B} \) field:

- If \( \vec{B} \) uniform \( \rightarrow \)
  - Charges in wire experience upward force \( q \vec{v} \times \vec{B} \)
  - So two opposite ends acquire charge

Evaluate line integral of force \( \int \vec{F} \cdot d\vec{s} \) around loop:

- Only two ends contribute since \( \vec{F} \) is parallel to \( d\vec{s} \) \( \rightarrow \) result in equal+opposite contributions

\[ \int \vec{F} \cdot d\vec{s} = 0 \]

If \( \vec{B} \) nonuniform \( \rightarrow \) two ends now do not cancel:

\[ \int \vec{F} \cdot d\vec{s} = q \vec{v} (B_1 - B_2) \omega \]

\[ \Rightarrow \text{EMF} \quad \mathcal{E} = \frac{1}{q} \int \vec{F} \cdot d\vec{s} = \vec{v} (B_1 - B_2) \omega \]

Induced current \( I = \frac{\mathcal{E}}{R} \)
Relate emf $\mathcal{E}$ to magnetic flux $\Phi_m = \int \mathbf{B} \cdot d\mathbf{a}$

In this example $d\Phi_m = -(B_1 - B_2) \omega \, v \, dt$

\[
\mathcal{E} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{a}
\]

\[\text{Lenz's Law (\Theta sign, physically very important)}
\]
\[\text{acts to oppose the change in flux}\]

Faraday:
\[\nabla \text{uniform } \mathbf{B} \rightarrow \mathcal{E}\]

\[\text{pull loop, get } I_{\text{ind}} \text{ (clockwise)}\]

\[\nabla \text{ pull magnet, not loop, now no } \mathcal{E} \times \mathbf{B}, \text{ but still identical } I_{\text{ind}}\]

\[\text{decrease strength of } \mathbf{B}. \text{ Also no } \mathcal{E} \times \mathbf{B}, \text{ but still get induced current } I\]
all are encoded by \[ E = -\frac{d}{dt} \int B \cdot d\mathbf{a} \]

Note that since \[ E = \frac{1}{2} \oint \mathbf{E} \cdot d\mathbf{s} = \oint \mathbf{E} \cdot d\mathbf{s} = \oint \mathbf{E}_{nc} \cdot d\mathbf{s} \]

(source of non-conservative \( \mathbf{E} \) field!
(not static field, for which \( \oint \mathbf{E} \cdot d\mathbf{s} = 0 \))

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**Example:**

Consider \( \mathcal{O} \mathbf{B} \) (coil)

- Increase \( \mathbf{B} \) \( \rightarrow \) \( \text{Ind} \) \( \rightarrow \) (opposes change in \( \mathbf{B} \))

Circulating \( \mathbf{E} \) field \( \mathbf{E}_{nc} \) set up in all space where there is a changing flux of \( \mathbf{B} \).

- Induced \( \mathbf{E} \) field in clockwise direction

Note: never could get this \( \mathbf{E} \) from static charges

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\( \text{Ind} \) \( \rightarrow \) \( \text{Ind} \) \( \rightarrow \) (signs: R.H. rule)

Circulation of \( \mathbf{E} \) + direction of \( d\mathbf{a} \)
Signs also understood by Lenz's law: induced emf is in a direction to oppose the change that produces it.

In our loop example:

- Increase $\vec{B}$ → $\vec{E}_{\text{ind}}$ (Find)
- Decrease $\vec{B}$ → $\vec{E}_{\text{ind}}$ (Find)

Back to

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

$$\Rightarrow \int \nabla \times \vec{E} \cdot d\vec{a} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

$$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

**Differential Statement of Faraday's Law**

Changing $\vec{B}$ field with time → source of $\vec{E}$ field.
Examples: categories by ways to change flux:

- change $\vec{B}$
- change magnitude of area $\vec{A}$
- change direction of $\vec{A}$

"Motional emf"

Motional emf examples:

1. Rotating coil in uniform $\vec{B}$ field, angular velocity $\omega$, area $A$, $N$ turns

$$\Phi_m = NBA \sin(\omega t + \alpha)$$

$$\Rightarrow E = -\frac{d\Phi_m}{dt} = -NBA \omega \cos(\omega t + \alpha)$$

2. Sliding bar in uniform $\vec{B}$ field:

$$\vec{E}_m = \vec{B} \cdot \vec{A} = BW \vec{x}$$

$$\Rightarrow E = -\frac{d\Phi_m}{dt} = -BWv$$

$$I_{ind} = \frac{E}{R} = \frac{BWv}{R}$$

Force $\vec{F} = I\vec{I} \times \vec{B}$

$$|\vec{F}| = IwB = \frac{B^2 w^2 v}{R}$$

direction: opposite motion

$\vec{v} \times \vec{B}$ force
Charging B example:

Long solenoid, $N$ turns/length, current $I(t)$, circular cross section, radius $b$

\[
E = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{a} = -\frac{d}{dt} \left( \mu_0 n I \pi b^2 \right) = -\mu_0 n \left( \frac{dI}{dt} \right) \pi b^2
\]

This is just our coil example from earlier.

\[\begin{array}{c}
\text{increasing } I(t) \rightarrow \text{Ind} \Rightarrow \text{Circ } \mathbf{B} \text{ Ind} \\
\text{decreasing } I(t) \rightarrow \text{Ind} \Rightarrow \text{Circ } \mathbf{B} \text{ Ind}
\end{array}\]