last time: currents + Ohm's law

today: basic circuits w/ resistors and capacitors
Sections 4.7 - 4.10

Circuit rules

1. $V = IR$

$\rightarrow R$

$\rightarrow I$

※ $V$ here is voltage drop
※ Positive charge direction is $I > 0$

e.g. $\rightarrow I > 0 \rightarrow e$

2. $\sum_{n=1}^{N} I_n = 0$ with $I$ arrow drawn as shown

3. The sum of potential differences around a closed loop is zero.

$\oint \mathbf{A} \cdot \mathbf{E} = 0$ assumed.

4. Batteries give a potential jump independently of the amount of current

$\Sigma \mathcal{E}$

Example

Find the equivalent resistance of $\overline{R_1 \parallel R_2}$

Ans.

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

Total voltage drop: $I (R_1 + R_2) = IR \Rightarrow R = R_1 + R_2$
Example
Find the equivalent resistance of

\[ R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} \]

since the voltage drop must be the same,

\[ IR = I_1 R_1 = I_2 R_2 \]
\[ I_1 + I_2 = I \]
\[ \therefore \frac{IR}{R_1} + \frac{IR}{R_2} = I \]
example

Find the current through resistor $R_2$

\[ E - I_1 R_1 + I_2 R_3 = 0 \]
\[ I_1 R_1 - I_2 R_2 = 0 \]
\[ I_1 = I_2 \frac{R_2}{R_1} \rightarrow I = I_2 \frac{R_2}{R_1} + I_2 \]
\[ -E + I_2 R_2 + I_2 \left( 1 + \frac{R_2}{R_1} \right) R_3 = 0 \]

\[ I_2 = \frac{E}{R_2 + R_3 \left( 1 + \frac{R_2}{R_1} \right)} \]

ans

method 1
Method 2

\[ I = \frac{E}{R_3 + R_4} \]

where \( R_4 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \)

\[ I_2 R_2 = I R_4 \]

\[ \Rightarrow I_2 = \frac{E}{R_3 + R_4} \frac{R_4}{R_2} = \frac{E}{1 + \frac{R_3}{R_4}} \frac{1}{R_2} \]

\[ = \frac{E}{1 + \frac{1}{R_3} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \frac{1}{R_2} \]

\[ = \frac{E}{R_2 + R_3 \left( 1 + \frac{R_2}{R_1} \right)} \]

Example

Find current \( I_1 \).
**Method 1**

Symmetry

\[ I_3 = 0 \]

\[ I_1 \]

\[ \Rightarrow I_1 \cdot 2R = E \]

\[ I_1 = \frac{E}{2R} \]

**Method 2**

\[ I_1 - I_3 R - I_4 R = 0 \]

\[ I_4 = I_1 - I_3 \]

\[ (I_1 + I_3) R - (I_4 - I_3) R + I_3 R = 0 \]

\[ -E + I_1 R + (I_1 + I_3) R = 0 \]

\[ I_3 = \frac{E}{R} - 2I_1 \]

\[ I_4 = I_1 - \left( \frac{E}{R} - 2I_1 \right) = 3I_1 - \frac{E}{R} \]
using \( \star \), we find

\[
(I_1 + \frac{\varepsilon}{R} - 2I_1) - (2I_1 - \frac{\varepsilon}{R} - \frac{\varepsilon}{R} + 2I_1) + \frac{\varepsilon}{R} - 2I_1 = 0
\]

\[
(-1 - 5 - 2) I_1 + 4 \frac{\varepsilon}{R} = 0
\]

\[
I_1 = \frac{1}{2} \frac{\varepsilon}{R}
\]

Section 4.11

Suppose the switch is closed at \( t=0 \). Find \( I(t) \)

answer

\[
Q = CV
\]

\[
I = -\frac{dQ}{dt} \text{ since if } \frac{dQ}{dt} < 0 \text{ \( I \geq 0 \) in the direction that is defined in the diagram.}
\]

\[
-V + IR = 0
\]

\[
\Rightarrow \frac{Q}{C} = IR
\]

\[
\frac{Q}{C} = -\frac{dQ}{dt} R \Rightarrow \int \frac{dQ}{Q} = -\int \frac{dt}{RC}
\]

\[
\ln \left( \frac{Q(t)}{Q_0} \right) = -\frac{t}{RC}
\]

\[
Q(t) = Q_0 e^{-\frac{t}{RC}}
\]
Note that $I(0) = \frac{Q_0}{RC}$.

Hence, near $t=0$, current flows as if there were a constant voltage of $\frac{Q_0}{C}$. 

\[ I(t) = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-\frac{t}{RC}} \]