Topics: Electric Currents (Purcell, Ch. 4)

Start of the study of charges in motion

\[ I = \text{charge/time} \quad \text{units} \quad \text{SI: Ampere} \quad A = \frac{C}{s} \quad \text{cgs: esu/s} \]

Focus for now on the idea of a steady current \( \rightarrow \) one with uniform flow.

Current in a wire \( \rightarrow \) current density

\[ \vec{J} \rightarrow \left[ \frac{\vec{J}}{m^2} \right] = \frac{A}{m^2} \]

Model of current flow: particles of charge \( q \), velocity \( \vec{v} \)

\[ \text{# density} \quad \rho = \text{particles/unit volume} \]

**Question:** how many particles flow past a patch of area \( \vec{a} \cdot \vec{v} \) in \( \text{time} \; \Delta t \)?

Consider the average rate:

\[ I_a = \left[ \frac{q}{\Delta t} \right] = \frac{n q \vec{a} \cdot \vec{v}}{\Delta t} \]

If many species:

\[ I_a = \sum i q_i \vec{a} \cdot \vec{v}_i = \vec{a} \cdot \sum \vec{v}_i q_i \]

\[ 

Define the current density $\mathbf{J}$ as $\mathbf{J} = \sum q_i \mathbf{u}_i$ in practice: $\mathbf{u}_i$ is average velocity for species $i$.

The current $I$ flowing through a surface $S$:

$$I = \int_S \mathbf{J} \cdot d\mathbf{a}$$

Charge conservation $\Rightarrow \int_S \mathbf{J} \cdot d\mathbf{a} = 0 \Rightarrow \nabla \cdot \mathbf{J} = 0$ by divergence theorem.

For non-steady currents:

$$\int_S \mathbf{J} \cdot d\mathbf{a} = -\frac{d}{dt} \int_V \rho \, dV \quad \Rightarrow \quad \int_V \nabla \cdot \mathbf{J} \, dV = -\frac{d}{dt} \int_V \rho \, dV$$

$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$

This is the statement of charge conservation.
Electrical conductivity and Ohm's law

Consider motion of charged particles in conductors as caused by external \( \vec{E} \) field:

- \( \vec{E} \) field \( \rightarrow \) Coulomb force
- Moves charged particles \( \rightarrow \) current

Many substances: Current density \( \vec{J} \) is linearly prop to \( \vec{E} \):

Ohm's Law \( \rightarrow \)[box]

\[ \vec{J} = \sigma \vec{E} \]

\( \sigma = \text{conductivity} \ [\Omega^{-1} \cdot \text{m}] \)

Can also introduce resistivity \( \rho = \frac{1}{\sigma} \)

Many materials \( \rightarrow \) \( \vec{J} \) is a scalar

\( \sigma \) can also have more complicated situations where \( \sigma \) is a tensor

"Isotropic"

Note: Ohm's law is an empirical relation, not a fundamental one.
- Fails in general for sufficiently strong \( \vec{E} \) fields
- Not all materials are "ohmic" even in weak fields
Consequence of Ohm's Law:

potential difference b/w points of conductor $\Delta \phi \equiv V$ (voltage)
is proportional to the current flow:

$\Rightarrow V = I R$

\[ [R] = \text{ohm } \Omega \]
resistance (depends on geometry and the conductivity of the material)

consider a rod of length $L$, cross-sectional area $A$
made out of material with conductivity $\sigma$

Run current through it: $J = I/A$

voltage: $V = EL$

$\Rightarrow R = \frac{V}{I} = \frac{EL}{JA} = \frac{L}{\sigma A} = \frac{PL}{A}$

this assumes uniform current density throughout
and that rod is surrounded by a nonconducting medium

\[ [\sigma] = \frac{1}{\Omega \text{ m}} \quad \text{or} \quad \frac{1}{\Omega \text{ cm}} \]

\[ [\rho] = \frac{1}{\Omega \text{ cm}} \]