Honors Question – Black Holes and Neutron Stars

In Friday's lecture, we learned that physicists think that black holes are mass singularities – an enormous mass concentrated in essentially a point. Since gravitational force increases as the distance decreases, it's natural to conclude that within some radius the gravitational force is so large that even light cannot escape. That radius is the event horizon.

Another interesting astronomical object is the neutron star, a stellar remnant which is composed of neutrons packed tightly together like marbles. In other words, a neutron star is essentially a giant neutron-only nucleus. It is held to a certain size by the Pauli Exclusion Principle of quantum mechanics, which says that two identical neutrons cannot occupy the same space.

The theory of neutron stars was developed years ago, and the result is that they have a very specific size and mass. The radius of a neutron star is $R = 12$ km (that's right, it's that small!). Since it is essentially a giant nucleus, it has mass density $\rho = 5 \times 10^{17} \text{kg/m}^3$, which puts its mass at around 1.4 – 2.0 solar masses. So, a neutron star is essentially a sun-sized star shrunk down to the smallest size it can be according to quantum physics.

An object can escape the gravitational pull of a large object if its kinetic energy exceeds the magnitude of the gravitational potential energy. Classically, kinetic energy is given by:

$$K = \frac{1}{2} m v^2$$

and the magnitude of gravitational potential energy is

$$U = G \frac{M m}{r}$$

where $M$ is the mass of the large object and $G = 6.7 \times 10^{-11} \text{m}^3/\text{kg s}^2$ is the gravitation constant.

(1) Show that, classically, a neutron star is not a black hole (this is also true relativistically).

(2) At what size would a neutron star become a black hole, according to classical physics?

(3) Suppose a black hole has the mass of the Sun. Calculate the radius of the event horizon using classical mechanics. (This isn't correct since one really needs to use relativistic mechanics, but it's in the right ballpark.)