Physics 207 Spring 2012 Practice Test 1

1. A fairly good model of a sprinter is that he accelerates out of the blocks with constant acceleration $a$, for time $t_a$, then finishes the race at constant speed. The total race is over a distance $d$.

a) Draw graphs of $x(t)$ and $v(t)$ for such a sprinter.

b) Derive an equation relating the total race time $t_f$ to the acceleration time $t_a$, acceleration $a$ and length of the race, $d$.

c) Usain Bolt holds the world record 100-m sprint time of 9.58 s. Assuming that Bolt accelerated for the first 5.00 seconds, calculate his acceleration $a$. 
2. A ball is kicked horizontally at the top of a hemispherical hill of radius $R$.

a) Find the minimum initial speed $v_0$ such that the ball never touches the sphere.

b) Find the minimum distance $x$ between the edge of the sphere and the point where the ball lands.
3. A simplified version of the Cavendish experiment is shown at right. A small mass $m$ hangs from a massless string of length $l$ and is attracted to a large, stationary mass $M$ by the force of gravity. The small mass is a distance $d$ from the large mass and the (very small) angle of deflection is $\theta$.

(a) Draw an expanded view of the small mass, exaggerating the angle $\theta$, and label the forces on it. (In other words, draw a free body diagram for $m$.)

(b) Derive an equation relating the gravitational constant $G$ to the given quantities and $g$. Feel free to use the small-angle approximation $\sin \theta \approx \theta$.

(c) In order to make the largest, and therefore most measurable deflection $\theta$, should $M$ be large or small? Should $d$ be large or small? Explain why.

(d) In the actual Cavendish experiment (which used a sensitive torsion spring, not this setup), $d = 0.4$ m, $M = 158$ kg, and $m = 0.73$ kg. Calculate the resulting deflection $\theta$. Do you think this angle was measurable in 1797?
4. In the previous problem, we solved for the gravitational constant $G$. But, as you know, Cavendish was more interested in the average density of the earth, $\rho$ than he was interested in $G$ itself. If he were using the simple system from problem 3, he'd have the following formula for his desired quantity in terms of the measured quantities, where $\theta$ is measured in radians and $R$ is the radius of the earth.

$$\rho = \frac{M}{\frac{4}{3} \pi R d^2 \theta}$$

(a) Being a good experimentalist, Cavendish is required to estimate the error on his measurement of the average density of earth. Derive the formula for the relative error in $\rho$, $\frac{\Delta \rho}{\rho}$, in terms of the relative error on all the other quantities.

(b) Make some reasonable guesses for the relative error on all the measured quantities $M$, $R$, $d$ and $\theta$, and calculate the resulting estimated relative error on $\rho$.

(c) Cavendish himself reported a result of $5.448 \pm 0.033$ times the density of water. How does his relative error compare to your estimate? Do you think Cavendish could have measured $\rho$ this accurately using the setup of problem 3?
5. A small block of mass $m_1$ lies on an inclined plane of mass $m_2$ which can slide without friction on the table. The static and kinetic friction coefficients between $m_1$ and $m_2$ are $\mu_s$ and $\mu_k$. A rope and frictionless pulley connect $m_2$ to a weight of mass $m_3$.

(a) Draw and label the forces acting on $m_1$.

(b) For the case where $m_3=0$, calculate the minimum value of $\mu_s$ such that $m_1$ does not slide.

(b) Now calculate the minimum value of $\mu_s$ such that $m_1$ does not slide in the case when $m_3 \neq 0$. 