Physics 207 Exam 3 (Hokin)
May 9, 2011

Name _____________________________
TA _______________________________

Problem 1 (hot air balloon) _______
Problem 2 (ping-pong ball cannon) _______
Problem 3 (250cc engine) _______
Problem 4 (rotating pitch generator) _______
Problem 5 (two facing speakers) _______
TOTAL _______

Constants, etc.

\[ c = 3.0 \times 10^8 \text{ m/s} \] (speed of light in vacuum)

\[ g = 9.8 \text{ m/s}^2 \] (near-Earth gravitational acceleration)

\[ T_0 = -273 \degree \text{ C} \] (absolute zero)

\[ 1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} \] (atomic mass unit)

\[ P_{\text{atm}} = 1 \times 10^5 \text{ Pa} \] (atmospheric pressure at sea level)

\[ v_{\text{sound}} = 343 \text{ m/s} \] (speed of sound at 20\degree \text{ C})

\[ N_A = 6.02 \times 10^{23} \text{ molecules/mole} \] (Avogadro's number)

\[ k = 1.38 \times 10^{-23} \text{ J/K} \] (Boltzmann's constant)

\[ \rho_{\text{air}} = 1.2 \text{ kg/m}^3 \] (mass density of air at STP)

\[ \rho_w = 1000 \text{ kg/m}^3 \] (mass density of water)
1. A hot air balloon consists of a large *envelope* of volume \( V \), along with a payload basket containing the passengers and a propane burner placed just below the opening of the envelope. The burner maintains the air inside the envelope at temperature \( T_h \) while the temperature of the surrounding air is \( T_c \). Air is free to enter and leave the envelope through the opening.

a) (5 pts) What is the absolute pressure inside the balloon when it is close to the ground? Provide a numerical answer.

b) (10 pts) Find the net force \( F_{net} \) on the balloon envelope, ignoring the payload, in terms of \( V \), \( g \), surrounding air density \( \rho_{air} \), \( T_c \) and \( T_h \).

c) (5 pts) Calculate \( F_{net} \) when \( V=3000 \, m^3 \), \( T_h=100 \, ^\circ C \), \( T_c=20 \, ^\circ C \) and \( \rho_{air}=1.2 \, kg/m^3 \). Approximately how many people can the balloon carry if the envelope plus basket plus burner plus fuel add up to 380 kg?
2. In lecture, I demonstrated a cannon in which a ping-pong ball is pushed out of an evacuated tube by ambient air pressure when the seal behind it is punctured.

a) (5 pts) A ping-pong ball has mass \( m = 2.7 \) g and diameter \( d = 40 \) mm. Calculate the force on the ball.

b) (5 pts) The length of the tube \( l = 3 \) m. Calculate the speed of the ball when it reaches the end of the tube.

c) (5 pts) Calculate the RMS speed of an air molecule. Assume that the average mass of an air molecule \( m = 29 \) u and \( T = 20 \) °C.

d) (5 pts) Compare your results in parts b) and c). Do you think the ball can reach the speed given in part b)? Explain your answer.
3. A two-stroke dirt bike engine has a single cylinder with displacement 250 cc which operates on the Otto cycle as shown.

a) (5 pts) Calculate the pressure $p_2$ at the end of the compression stroke. Assume that the gas inside the cylinder is a diatomic ideal gas.

b) (5 pts) Given $T_1 = 20 \, ^\circ C$, calculate the temperature of the compressed fuel before ignition, $T_2$.

c) (5 pts) If combustion raises the temperature of the gas to $T_3 = 2000 \, K$, calculate the heat input to the gas, $Q_h$.

d) (5 pts) The efficiency of an Otto engine is
\[ \eta = 1 - \frac{1}{r^{\gamma - 1}} \]
where $r = \frac{V_1}{V_2}$ is the compression ratio. Calculate the total work $W$ done by this engine during one full cycle and the power $P$ delivered by this engine at 6000 RPM (cycles per minute).
4. In lecture I demonstrated the Doppler effect by spinning a pitch generator taped inside a Nerf ball around my head on a string.

a) (10 pts) If the string length is \( r \) and I swing the ball at angular speed \( \omega \), determine the relative pitch difference \( \Delta f / f_0 \) heard by a distant listener, where \( f_0 \) is the stationary pitch.

b) (5 pts) Calculate \( \Delta f / f_0 \) when the ball is swung at 2 rotations per second and the string length is 1 m.

c) (5 pts) At what angular speed \( \omega \) must the ball revolve for there to be an octave (factor of two) between the lowest and highest pitch, i.e., \( f_{\max} = 2 f_{\min} \)? Is this possible? Explain.
5. A rather bad sound engineer makes the mistake of mounting two identical speakers facing each other a distance $l$ apart, as shown. The speakers emit the exact same sound.

a) (10 pts) Write expressions for the phases of the two waves, $\phi_1(x,t)$ and $\phi_2(x,t)$ in terms of wave number $k$, frequency $\omega$ and $l$. Make sure your equations ensure that the two waves always have the same phase at the sources, i.e. $\phi_1(-l/2,t)=\phi_2(l/2,t)$.

b) (5 pts) At what position $x$ do the two waves always add constructively, for any wavelength?

c) (5 pts) For a given wavelength $\lambda$, determine the positions $x_m$ where the sound level is minimum. Indicate the allowed values of $m$. 