Physics 208, Lecture 6

Today’s Topics

- More on Electric Potential
- Calculate Electric Field from Potential
- Capacitance (Ch. 26.1-3)
- Capacitors and Capacitance
- Calculating Capacitance for parallel-plate, cylindrical, spherical capacitors.
- Combinations of capacitors
- Hope you have previewed!

About Exam 1

- When and where
  - Wednesday Feb. 22nd 5:30-7:00 pm
  - (room to be announced)
- Format
  - Closed book
  - One 8x11 formula sheet allowed, must be self prepared, no photo copying/download-printing of solutions, lecture slides, etc.
  - 20-25 multiple choice questions
  - Bring a calculator (but no computer). Only basic calculation functionality can be used.
  - Bring a B2 pencil for Scantron.
- Special requests:
  - Have to be approved. Deadline is 6pm tomorrow (Feb 10th.)
  - All specially arranged tests (e.g. those at alternative time) are held in our 202 labs. (for approved requests only)

Chapters Covered

- Chapter 23: Electric Fields
- Chapter 24: Gauss’s Law
- Chapter 25: Electric Potential
- Chapter 26: Capacitance

I will not post past/sample exams as none that I can find are representative. Often those can be misleading.

I will use next Thursday’s lecture to review for the test. (and will show a few sample test questions to help you get familiar with the test style)

Review: Electric Potential Difference

- Electric Potential Energy: \( q \) in a Generic E. Field
  \[
  \Delta U = U_B - U_A = -q \int_A^B \mathbf{E} \cdot d\mathbf{s} = q \Delta V
  \]

- Electric Potential Difference
  \[
  \Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = V_B - V_A
  \]
Understand a Battery (I)

- What is a 1.5V battery?
  - Chemical process maintains a charge distribution, such that $V_+ - V_- = 1.5V$, regardless of shape.
  - Electric energy is stored in the E field.

Understand a Battery (II)

- When connected to a load
  - Electron flow from negative side to positive side.
  - In the process, the 1.5V potential difference is maintained.
  - Kinetic energy acquired by each electron 1.5 eV.
  - This energy is converted into heat, light etc.
  - Chemical energy $\rightarrow$ electric potential energy $\rightarrow$ load

Calculate Electric Field From The Electric Potential

- Three ways to calculate the electric field
  - Superposition $E = \sum E_i$
  - Gauss’ s Law
  - From the gradient of electric potential $\nabla V$
  - Formulism $\Delta V = \int E \cdot ds$

$\int dV = -E \cdot ds = -E_x dx - E_y dy - E_z dz$

$E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$ or $E = -\nabla V$

Review Example (Lecture 5): Uniformly Charged Ring

- For a uniformly charged ring, show that the potential along the central axis is $V = \frac{kQ}{\sqrt{x^2 + a^2}}$

Solution

$V = \int \frac{kQ}{\sqrt{x^2 + a^2}}$

$= \frac{kQ}{\sqrt{x^2 + a^2}} \int dq$

$= \frac{kQ}{\sqrt{x^2 + a^2}} \cdot \frac{a}{2}$

$= \frac{kQa}{2} \frac{1}{\sqrt{x^2 + a^2}}$

Uniformly Charged Ring: Electric Field

- Find the electric field along the central axis.
  - Approach 1: Superposition. (Example 23.7 in text)
    \[ dE_x = dE \cos \theta = \frac{k \lambda x}{r^2} \]
    \[ E_x = \int dE_x = \frac{k x Q}{r^2} \left( \frac{k x Q}{r^2} \right) \]
    \[ E_x = 0 \text{ due to symmetry} \]
  - Approach 2: derivative of potential
    \[ E_x = -\frac{\partial V}{\partial x} = \frac{k x Q}{(x^2 + a^2)^{3/2}} \]

Exercise: Parallel Plates

- Find the potential difference between the two large conductor plates of area A and separation d
  \[ \Delta V = \frac{Q d}{\epsilon_0 A} \]
  - Answer
    \[ \Delta V = Q d / (\epsilon_0 A) \]
  - Note: \( \Delta V \) is proportional to Q

Capacitors

- A generic capacitor:
  - Two conductors oppositely charged
    \[ \Delta V \propto Q \]
  - Capacitor are very useful devices:
    - Timing control, noise filters, energy buffer, frequency generator/selector/filter, sensors, memories...

Demo: Charging A Pair of Parallel Conductors

- Uncharged
- Charging
  \[ \Delta V = V_+ - V_- \]
Capacitance

- $\Delta V \propto Q \Rightarrow Q = C \Delta V \Rightarrow C$ is called capacitance
- $C = \frac{Q}{\Delta V}$: amount of charge per unit of potential diff.
- Unit: Farad (F) = 1 Coulomb/Volt
- Parallel-plate: $C = \varepsilon_0 \frac{A}{d}$
- Cylindrical and Spherical: see examples in text
  - Cylindrical: $C = \frac{\varepsilon_0 \lambda}{\ln(b/a)}$
  - Spherical: $C = \frac{\varepsilon_0 \lambda}{\ln(b/a)}$

Combinations of Capacitors In Series

- Effective Capacitance
  - $C = \frac{Q}{\Delta V}$
  - $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$
  - Note: $C_{\text{series}}$ always < $C_i$

Combinations of Capacitors In Parallel

- Effective Capacitance
  - $C = \frac{Q}{\Delta V}$
  - $\frac{1}{C_{\text{parallel}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$
  - Note: $C_{\text{parallel}}$ always > $C_i$

Quick Quiz/exercise: Combination of Capacitors

- What is the effective capacitance for this combination?
  - $(C_1 = 1\mu F, C_2 = 2\mu F, C_3 = 3\mu F)$
  1. $C = 6\mu F$
  2. $C = 3\mu F$
  3. $C = 1.5\mu F$
  4. None of above
### Charging A Capacitor

- **Electric potential energy gained:**

\[
U = \int du = \int (-\Delta V)(-dq) - \int_0^Q \frac{dU}{C}dq = \frac{1}{2} \frac{Q^2}{C}
\]

- After charging the capacitor stores potential energy:

\[
U = \frac{1}{2} \frac{Q^2}{C}
\]

### Discharging A Capacitor

- **Potential energy released:**

\[
U = \int dU = \int \Delta V(-dq) - \int_0^Q \frac{dU}{C}dq = \frac{1}{2} \frac{Q^2}{C}
\]

- the originally charged capacitor has potential energy:

\[
U = \frac{1}{2} \frac{Q^2}{C}
\]