Physics 208, Lecture 6

Today’s Topics

- More on Electric Potential
- Calculate Electric Field from Potential
- Capacitance (Ch. 26.1-3)
  - Capacitors and Capacitance
  - Calculating Capacitance for parallel-plate, cylindrical, spherical capacitors.
- Combinations of capacitors

- Hope you have previewed!
About Exam 1

- When and where
  - Wednesday Feb. 22\textsuperscript{nd} 5:30-7:00 pm
  - (room to be announced)

- Format
  - Closed book
  - One 8x11 formula sheet allowed, must be self prepared, no photo copying/download-printing of solutions, lecture slides, etc.
  - 20-25 multiple choice questions
  - Bring a calculator (but no computer). Only basic calculation functionality can be used.
  - Bring a B2 pencil for Scantron.

- Special requests:
  - Have to be approved. Deadline is 6pm tomorrow (Feb 10\textsuperscript{th}.)
  - All specially arranged tests (e.g. those at alternative time) are held in our 202 labs. (for approved requests only)
Chapters Covered

- Chapter 23: Electric Fields
- Chapter 24: Gauss’ s Law
- Chapter 25: Electric Potential
- Chapter 26: Capacitance

I will not post past/sample exams as none that I can find are representative. Often those can be misleading.

I will use next Thursday’s lecture to review for the test. (and will show a few sample test questions to help you get familiar with the test style)
Review: Electric Potential Difference

- Electric Potential Energy: $q$ In a Generic E. Field

$$\Delta U = U_B - U_A = -q \int_{A}^{B} \mathbf{E} \cdot ds = q \Delta V$$

- Electric Potential Difference

$$\Delta V \equiv \frac{\Delta U}{q} = -\int_{A}^{B} \mathbf{E} \cdot ds = V_B - V_A$$
Understand a Battery (I)

- What is an 1.5V battery?

- Chemical process maintains a charge distribution, such that $V_+ - V_- = 1.5V$, regardless of shape.

- Electric energy is stored in the E field.
Understand a Battery (II) In-Use

- When connected to a load

- Electron flow from negative side to positive side.
- In the process, the 1.5V potential Diff. is maintained.
- Kinetic energy acquired by each electron 1.5 eV.
- This energy is converted into heat, light etc.
- Chemical energy $\rightarrow$ electric potential energy $\rightarrow$ load
Calculate Electric Field From The Electric Potential

- Three ways to calculate the electric field
  - Superposition $\mathbf{E} = \sum \mathbf{E}_i$
  - Gauss’ s Law
  - From the gradient of electric potential $\Rightarrow$
    - Formulism
      $$\Delta V = -\int_A^B \mathbf{E} \cdot \mathbf{ds}$$
      $$dV = -\mathbf{E} \cdot \mathbf{ds} = -E_x \, dx - E_y \, dy - E_z \, dz$$
      $$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \quad \text{or} \quad \mathbf{E} = -\nabla \cdot \mathbf{V}$$
Review Example (Lecture 5) : Uniformly Charged Ring

For a uniformly charged ring, show that the potential along the central axis is

\[ V = \frac{k_e Q}{\sqrt{x^2 + a^2}} \]

Solution

\[ V = \int \frac{k_e \, dq}{r} = \int \frac{k_e \, dq}{\sqrt{x^2 + a^2}} = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = Q \]
Uniformly Charged Ring: Electric Field

- Find the electric field along the central axis.

- Approach 1: Superposition. (Example 23.7 in text)

\[ dE_x = dE \cos \theta = \frac{k_e dq}{r^2} \frac{x}{r} \]

\[ E_x = \int dE_x = \frac{k_e x Q}{r^3} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}} \]

\[ E_\perp = 0 \text{ due to symmetry} \]

- Approach 2: derivative of potential

\[ E_x = -\frac{\partial V}{\partial x} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}} \]
Exercise: Parallel Plates

Find the potential difference between the two large conductor plates of area \( A \) and separation \( d \)

\[ \Delta V = \frac{Qd}{\varepsilon_0 A} \]

**Answer**

\( \Delta V = \frac{Qd}{\varepsilon_0 A} \)

Note: \( \Delta V \) is proportional to \( Q \)

See board

Realistic case
Capacitors

- A generic capacitor:

- Capacitors are very useful devices:
  - Timing control, noise filters, energy buffer, frequency generator/selector/filter, sensors, memories...

\[ \Delta V \propto Q \]
Demo: Charging A Pair of Parallel Conductors

\[ \Delta V = V_+ - V_- \]
Capacitance

- $\Delta V \propto Q \rightarrow Q = C \Delta V \rightarrow C$ is called capacitance
- $C = Q/\Delta V$: amount of charge per unit of potential difference.
  - Unit: Farad (F) = 1 Coulomb/Volt
  - Parallel-plate: $C = \varepsilon_0 A/d$
  - Cylindrical and Spherical: see examples in text

- **Cylindrical:**
  \[ C = \frac{\ell}{2k_e \ln(b/a)} \]

- **Spherical:**
  \[ C = \frac{ab}{k_e (b - a)} \]
Combinations of Capacitors In Series

Charge conservation: \( Q_1 = Q_2 (=Q) \)

Effective Capacitance
\[
C = \frac{Q}{\Delta V} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}
\]

\[
\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots
\]

Note: \( C_{\text{series}} \) always < \( C_i \)
Combinations of Capacitors In Parallel

\[ C_1 \Delta V_1 = Q_1 \]
\[ C_2 \Delta V_2 = Q_2 \]

\[ \Delta V_1 = \Delta V_2 = \Delta V \quad \text{(why?)} \]

Effective Capacitance
\[ C = \frac{Q}{\Delta V} \Rightarrow C = C_1 + C_2 \]

\[ C_{\text{parallel}} = C_1 + C_2 + C_3 + \ldots \]

Note: \( C_{\text{parallel}} \) always > \( C_i \)
Quick Quiz/exercise: Combination of Capacitors

What is the effective capacitance for this combination? 
\((C_1=1\,\mu F, C_2=2\,\mu F, C_3=3\,\mu F)\)

1. \(C=6\,\mu F\)
2. \(C=3\,\mu F\)
3. \(C=1.5\,\mu F\)
4. None of above
Charging A Capacitor

Electric potential energy gained:

\[
U = \int du = \int (-\Delta V)(-dq) = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}
\]

After charging the capacitor stores potential energy:

\[
U = \frac{1}{2} \frac{Q^2}{C}
\]
Discharging A Capacitor

Potential energy released:

\[ U = \int dU = \int \Delta V (-dq) = \int_0^0 -\frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} \]

the originally charged capacitor has potential energy:

\[ U = \frac{1}{2} \frac{Q^2}{C} \]