Physics 202, Lecture 5

Today’s Topics

- Electric Potential (Ch. 25-Part I)
  - Electric Potential Energy & Electric Potential
  - Electric Potential And Electric Field
  - Electric Potential For Various Charge Distributions
    - Point charges
    - Continuous charges

.randn from preview:
  - Conservative force, electric energy, electric potential difference, voltage,…
  - $\mathbf{E} \leftrightarrow \mathbf{V}$ relationship, equipotential lines,…
  - Electric potential for a charge distribution…

Review: Electric Field and Electric Force

Electric Field is a form of matter. It carries energy (Next week)

Potential Energy (Phy201 Review)

- Ch-8: path independent work $\rightarrow$ conservative force.
- e.g. Gravitational Force is a conservative force (Ch-13):

  \[ F_{12} = -G \frac{m_1 m_2}{r^2} \]

  \[ W = \int_{j=1} F \cdot dl = G \frac{m_1 m_2}{r} \]

  \[ U = - \frac{G m_1 m_2}{r} \]

  ➢ Gravitational Potential energy:
    - path independent!

  ➢ Electric Force:

    \[ F_{12} = k \frac{q_1 q_2}{r^2} \]

    \[ U = k \frac{q_1 q_2}{r} \]

Electric Potential Energy

- Like gravitational force, electric force is also a conservative force
  $\rightarrow$ A potential energy can be defined.
- Electric potential energy between two point charges:
  \[ U = k \frac{q_1 q_2}{r} \]

  ➢ $U$ is a scalar quantity

  ➢ $U=0$ @ $r=\infty$ (convenient convention)

  ➢ $U>0$: between like-sign charges
  ➢ $U<0$: between opposite charges

  ➢ SI unit: Joule (J)

- Electric potential energy for system of multiple charges/charge distributions:
  \[ U = \sum \text{all combination of pairs.} \]
Example: Three Charge system

- What is the work required to assemble the three charge system as shown? \((q_1=q_2=q_3=Q)\)
  
  Answer: \(k_e 3Q^2/a\) (see board)

- Quiz: What if \(q_1=q_2=Q\) but \(q_3=-Q\) ?
  
  Answer: \(-k_e Q^2/a\)

Electric Potential Energy For Charge In An Electric Field

- Charge \(q\) is subject an electric force in electric field \(E\)
  
  \(F=qE\)

- Work done by electric force:
  
  \[
  W = \int_{A}^{B} F \cdot ds = q \int_{A}^{B} E \cdot ds = -\Delta U
  \]

  \[
  \Delta U = U_B - U_A = -q \int_{A}^{B} E \cdot ds
  \]

  \(\) independent of \(q\)

Electric Potential Difference

- Electric Potential Energy: \(q\) In a Generic E. Field
  
  \[
  \Delta U = U_B - U_A = -q \int_{A}^{B} E \cdot ds = q \Delta V
  \]

- Electric Potential Difference
  
  \[
  \Delta V = \frac{\Delta U}{q} = -\int_{A}^{B} E \cdot ds = V_B - V_A
  \]

Properties of Electric Potential Difference

- It is defined upon the fact that the electric force is a conservative force.
- It is associated to the source field only and is independent of test charge.
- It has a unit: \(J/C = Volt (V)\)
- It is commonly called as just Potential, but it is meaningful only as potential difference \(V_B - V_A\).
- Usually a convenient point (remote, earth..) is chosen as "ground" \(\Delta V=V-(V_A=0)=V\)
- It is a scalar quantity. (No vector operation necessary!)
- \(\Delta U=q\Delta V\)
Exercise 1:
Electric Potential and Point Charges

- In the field surrounding a point charge, find the potential difference \( V_B - V_A \).

**Answer:**
\[ V_B - V_A = k_e \left( \frac{q}{r_B} - \frac{q}{r_A} \right) \]
(See board)

Exercise 2: Potential In Uniform E. Field

- In the uniform electric field shown, find E. potential at points: B, C, D, G.
- If a charge +q is placed at B, what is its potential energy \( U_B \)? (assuming \( U_A = 0 \)).
- If a charge -q is at B, what is \( U_B \)?
- If a negative charge -q is initially at rest at G, will it move to A or B?
- What is the kinetic energy when it reaches A?

Visualization of Electric Potential
Equipotential Lines

- Field lines always point towards lower electric potential.
- Field lines and equal-potential lines are always at a normal angle.
- In an electric field:
  - +q is always subject a force in the same direction of field line. (i.e. towards lower V)
  - -q is always subject a force in the opposite direction of field line. (i.e. towards higher V)

A Picture to Remember

\[ F = qE \]
\[ \Delta U = q \Delta V \]
\[ W = \Delta U = \int E \cdot ds \]
Electric Potential For Continuous Charge Distribution

For finite charge distribution, it is common to set $V=0$ at infinite.

$$V = \sum k_e \frac{dq_i}{r_i} = k_e \int \frac{dq}{r}$$

If the charge distribution is known, $V$ can be calculated simply by scalar integral.

$$V_i = k_e \frac{dq_i}{r} \quad (V=0 \at \infty)$$

Example: Uniformly Charged Ring

For a uniformly charged ring, show that the potential along the central axis is

$$V = \frac{k_e Q}{\sqrt{x^2 + \alpha^2}}$$

Solution

$$V = \int \frac{k_e dq}{r} = \frac{k_e}{\sqrt{x^2 + \alpha^2}} \int dq = \frac{Q}{\alpha}$$

Example: Uniformly Charged Spherical Shell

For uniformly charged spherical shell.

Again, use:

$$\Delta V = \int_E \mathbf{E} \cdot dA = V_b - V_a$$

Tip:

$V$ is the same inside $E=0$ region

$$E = 0 \quad r < R$$

$$E = \frac{k_e Q}{r^2} \quad r > R$$

Example: Uniformly Charged Sphere

Show that for a uniformly charged sphere, the electric potential is:

Hint: It is more convenient to use:

$$\Delta V = \int_E \mathbf{E} \cdot dA = V_a - V_b$$

from Gauss' s law:

$$E_r = \frac{k_e Q}{r^2} \quad r > R$$

$$E_r = \frac{k_e Q}{R^2} \quad r < R$$