Electric Fields and Gauss’s Law

http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html

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Review

- The electric force between two point-like charges is given by Coulomb’s Law.
- The electric field at a point in space is the total electric force per unit charge on a test charge at that point.
- The field may be visualized by field lines.
- The electric field of a collection of point charges in the vector sum of the electric fields of the individual charges.

Electric field of a continuous charge distribution

- Divide the charge distribution into small elements, each of which contains a charge $\Delta q_i$.
- Calculate the electric field due to one of these elements at point $P$.
- Evaluate the total field by summing the contributions of all the charge elements.

\[ F_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \]

The number of field lines leaving the positive charge equals the number terminating at the negative charge.

Charge densities

- Volume charge density: when a charge is distributed evenly throughout a volume
  \[ \rho = \frac{Q}{V} \text{ with units C/m}^3 \]  
  “rho”

- Surface charge density: when a charge is distributed evenly over a surface area
  \[ \sigma = \frac{Q}{A} \text{ with units C/m}^2 \]  
  “sigma”

- Linear charge density: when a charge is distributed along a line
  \[ \lambda = \frac{Q}{\ell} \text{ with units C/m} \]  
  “lambda”

\[ E(x) = \frac{1}{e} \left( \sum_i k_e \frac{dq_i}{r_{i}^2} \hat{r}_i \right) \]

\[ dq(x') = \rho(x')dV' = \sigma(x')dA' = \lambda(x')dx' \]
Problem solving strategies

- Analyzing a group of individual charges:
  - Use the superposition principle, find the fields due to the individual charges at the point of interest and then add them as vectors to find the resultant field.
  - Be careful with the manipulation of vector quantities.
- Analyzing a continuous charge distribution:
  - The vector sums for evaluating the total electric field at some point must be replaced with vector integrals.
  - Divide the charge distribution into infinitesimal pieces, calculate the vector sum by integrating over the entire charge distribution.
- Symmetry:
  - Take advantage of any symmetry to simplify calculations.

Field of a line charge

- To find the field of a uniformly charged rod, pick a coordinate system aligned with the rod and consider the charge dq in a small element of rod.
- Compute the contribution to the field at a point $x$ from an element at $x'$.

$E_y(x) = \int_a^b \frac{k_e \lambda (x' - x)^2 + y^2)^{3/2}}{y \sqrt{(x' - x)^2 + y^2} \sqrt{(b - x)^2 + y^2} \sqrt{(a - x)^2 + y^2}} \, dx'$

$E_y(x) = \frac{k_e \lambda}{y} \left( \frac{x' - x}{(b - x)^2 + y^2} - \frac{a - x}{(a - x)^2 + y^2} \right)$

For $a \to -\infty, b \to \infty$

Field of an infinite uniform line charge

$E(x) = \frac{2k_e \lambda}{\rho} \hat{\rho}$

By symmetry, the field of an infinitely long line charge points in the radial direction. The magnitude is proportional to the inverse of radial distance.

Compute the components

- With the help of integral calculus, we can compute the components of the total electric field.
- The radial $y$-component for an infinitely long line charge is inversely proportional to distance from the line charge.
A discrete model

- A JAVA applet is used to find the field of a finite collection of equal charges approximating a uniformly charged rod.
- The field lines flow away from the charges. (A few errant field lines crossing other lines are errors in the calculation.)
- Near the line charge, the field appears as if the collection had infinite extent.

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Another example

- To find the field on the axis of a uniformly charged disk, divide the disk into rings of infinitesimal width dr.
- Find the field at the point x due to each ring and then sum the field over rings converting the sum to an integral.
- For disk radius $R \gg x$, the field is that of an “infinite” plane of charge - uniform and normal to the disk.

Motions of charges

- A electron is projected horizontally into a uniform electric field.
- The electron undergoes a downward acceleration. The electron charge is negative, so the acceleration is opposite the direction of the field.
- The motion is parabolic while between the plates.

\[
\begin{align*}
F &= qE = -eE = m_e a \\
\mathbf{a} &= -\frac{e}{m_e} \mathbf{E} \\
\mathbf{v} &= \mathbf{v}_0 + \mathbf{a}t \\
x &= x_0 + v_0 t + \frac{1}{2} a t^2
\end{align*}
\]

An electron accelerator

- A simple accelerator makes use of two uniformly charged plates of metal with opposite charge.
- An electron released from the negatively charged surface is repelled by negative charge and attracted by positive charge so accelerates.
An electron gun with a hot filament electron source

- Heating a wire can cause electrons to escape the metal of the wire.
- Heating is done by pushing electrons through the wire as an electrical current. Their microscopic collisions within the metal provide the thermal energy.

Cathode ray tube

- A CRT uses a hot wire electron source and simple accelerator to form an electron beam.
- Electromagnets and electrostatic deflection plates serve to point the beam at a phosphor coated screen creating light.
- The beam may be turned on and off and its position on the screen changed rapidly.