Chapter 36

Image Formation
Light from distant things

- We learn about a distant thing from the light it generates or redirects.
- The lenses in our eyes create images of objects our brains can process.
- This chapter concerns imaging science and engineering.
Light is a wave and interferes with itself when interacting with, for example, an aperture. For apertures large compared to a wavelength, diffraction spreads light out. (See next chapter.)

Geometrical optics neglect diffraction and treats light energy as moving along straight rays in any uniform medium.

Light from different source points on an object generally is mixed together when detected at some surface. Light reflected by you bathes me fairly uniformly.
A small (but not too small!) aperture creates an approximate one to one map between object source points and points of illumination.

The faithful representation of the object by light on a downstream surface is called a real image.

In a pin hole camera, a light detector (film or CCD) at an image plane captures a 2D image.

Smaller hole=>sharper but dimmer image, sharpness diffraction limited.
When light is incident upon a surface, some is absorbed, some appears with *specular* reflection angle equal to the incident angle (relative to the normal), and some is diffusely reflected.

A polycrystalline polished surface produces diffuse reflection.

Polished metal produces principally specular reflection and is used for mirrors.
A planar mirror is usually made by evaporating metal on a glass sheet.

A planar mirror produces a virtual image to an observer. The reflected light appears to emerge from behind the mirror surface.

At any observation point, the light from different object points is mixed. A planar detector will not capture an image of objects but the reflected light is equivalent to the light from the object itself, just redirected so may used to create a real image.
Parabolic mirror

A parabolic shape is ideal for focussing all light from a distant source on axis to a point, the focal point. A parabolic cylindrical mirror is used to collect sunlight.

A concave spherical mirror is easier to construct and approximately parabolic.

Off axis, the focal “point” is smeared, an “aberration” called coma.
A spherical *convex* mirror produces a virtual image.

A spherical *concave* mirror can produce a real image. There is a common “image plane” where all the near axis light from any object point converges, is focused. There results a one-to-one map from object to image.

The location and size of the image depends on the distance and can be found by ray tracing.

Parallel rays along the optical axis are focused at the focal point, a distance $f$ from the mirror. The focal length of a spherical mirror is $f = R/2$.

The signed distance $p = x_o$ of the object plane and the signed distance $q = x_i$ of the image plane from the mirror are related to the focal length by the mirror equation. Positive = real, negative = virtual.

The magnification $M$ is the height of the image divided by the height of the object and is the image distance divided by the object distance, a negative value meaning the image is inverted.
A concave mirror of radius 20 cm. Find the image location for object distance $x_o = 40$ cm.

What is the magnification?

The image is real and inverted and smaller than the object.
Consider two transparent media having indices of refraction $n_1$ and $n_2$.

The boundary between the two media is a spherical surface of radius $R$.

Rays originate from the object at point $O$ in the medium with $n = n_1$ and converge to an image point in the medium with $n = n_2$.
We will consider the paraxial rays leaving O. All such rays are refracted at the spherical surface and focus at the image point, I.

The relationship between object and image distances can be given by

\[
\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}
\]
The side of the surface in which the light rays originate is defined as the front side.

The other side is called the back side.

Real images are formed by refraction in the back of the surface.

Because of this, the sign conventions for $q$ and $R$ for refracting surfaces are opposite those for reflecting surfaces.
Sign Conventions for Refracting Surfaces

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Positive When . . .</th>
<th>Negative When . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object location ($p$)</td>
<td>object is in front of surface (real object).</td>
<td>object is in back of surface (virtual object).</td>
</tr>
<tr>
<td>Image location ($q$)</td>
<td>image is in back of surface (real image).</td>
<td>image is in front of surface (virtual image).</td>
</tr>
<tr>
<td>Image height ($h'$)</td>
<td>image is upright.</td>
<td>image is inverted.</td>
</tr>
<tr>
<td>Radius ($R$)</td>
<td>center of curvature is in back of surface.</td>
<td>center of curvature is in front of surface.</td>
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Flat Refracting Surfaces

• If a refracting surface is flat, then R is infinite.

• Then \( q = -(n_2 / n_1)p \).

• The image formed by a flat refracting surface is on the same side of the surface as the object.

• A virtual image is formed.
Images Formed by Thin Lenses

- Lenses are commonly used to form images by refraction.
- Lenses are used in optical instruments.
  - Cameras
  - Telescopes
  - Microscopes
- Light passing through a lens experiences refraction at two surfaces.
- The image formed by one refracting surface serves as the object for the second surface.
The lens has an index of refraction \( n \) and two spherical surfaces with radii of \( R_1 \) and \( R_2 \).

- \( R_1 \) is the radius of curvature of the lens surface that the light of the object reaches first.

- \( R_2 \) is the radius of curvature of the other surface.

- The object is placed at point \( O \) at a distance of \( p_1 \) in front of the first surface.
Locating the Image Formed by a Lens, Image From Surface 1

- There is an image formed by surface 1.
- Since the lens is surrounded by the air, $n_1 = 1$.
- Write the lens equation for the first surface.
- If the image due to surface 1 is virtual, $q_1$ is negative; and it is positive if the image is real.

\[
\frac{n_1}{p_1} + \frac{n_2}{q_1} = \frac{n_2 - n_1}{R_1} = \frac{n - 1}{R_1}
\]
Locating the Image Formed by a Lens, Image From Surface 2

- For surface 2, $n_1 = n$ and $n_2 = 1$.

- The light rays approaching surface 2 are in the lens and are refracted into air.

- Use $p_2$ for the object distance for surface 2 and $q_2$ for the image distance.

\[
\frac{n_1}{p_2} + \frac{n_2}{q_2} = \frac{n_2 - n_1}{R_2}
\]

\[
\frac{n}{p_2} + \frac{1}{q_2} = \frac{1 - n}{R_2}
\]
Locating the Image, Surface 2

- The image due to surface 1 acts as the object for surface 2.
- Suppose the lens thickness is $t$.

The image due to surface 1 is real, so $I_1$ is to the right of the surface.
Lens-makers’ Equation

- If a virtual image is formed from surface 1, then \( p_2 = -q_1 + t \) where \( q_1 \) is negative and \( t \) is the thickness of the lens.

- If a real image is formed from surface 1, then \( p_2 = -q_1 + t \) where \( q_1 \) is positive.

- Combining the effect of the two surfaces and neglecting \( t \) yields the lens-makers’ equation.

\[
\frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}
\]

- It can be used to determine the values of \( R_1 \) and \( R_2 \) needed for a given index of refraction and a desired focal length \( f \).