Recall the series RC circuit.

If C is discharged and then a constant voltage V is suddenly applied, the charge on, and voltage across, C is initially zero. The charge ultimately reaches the value \( Q = CV \) and current ceases.

The characteristic time is \( RC \).
Consider an AC voltage applied to a capacitor.

Neglect any resistance in the wires. The charge follows the drive voltage.

The current is the time derivative of the charge and leads the drive voltage by 90 degrees.

The capacitive reactance $X_C$ is large at low frequency and small at high frequency.

\[ q(t)/C = \Delta v(t) = \Delta V_{\text{max}} \sin \omega t \]
\[ i_C(t) = \frac{dq}{dt} = -\Delta V_{\text{max}} \omega C \cos \omega t \]
\[ = X_C \Delta V_{\text{max}} \sin(\omega t + \frac{\pi}{2}) \]
\[ i_{C,\text{max}} = V_{\text{max}}/X_C \]

where
\[ X_C = \frac{1}{\omega C} \]
Capacitors in an AC Circuit

- A capacitor is connected between the terminals of a variable-frequency fixed-amplitude AC voltage source.

- Is there more current through the capacitor at low frequency or at high frequency?
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- The capacitive reactance $X_C$ is large at low frequency and small at high frequency. The current is small at low frequency and large at high frequency.
More About Capacitors in an AC Circuit

- The current is largest when the capacitor charge and voltage are small.
- The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.
- The current leads the voltage by 90°.
The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by $90^\circ$. The current and voltage phasors are at $90^\circ$ to each other.
The resistor, inductor, and capacitor can be combined in a circuit.

The current and the voltage in the circuit vary sinusoidally with time. Current conservation implies the current in each element is identical in amplitude and phase.
Equate the instantaneous drive voltage to the sum of voltage drops and use trig identities to expand. (I is the current amplitude, phi its phase.)

\[ V_0 \sin \omega t = IR \sin(\omega t + \phi) + IX_L \sin(\omega t + \phi - \frac{\pi}{2}) + IX_C \sin(\omega t + \phi + \frac{\pi}{2}) \]

\[
\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b \\
\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b
\]

\[ V_0 \sin \omega t = IR(\sin \omega t \cos \phi + \cos \omega t \sin \phi) \]
\[ + IX_L(\sin \omega t \sin \phi - \cos \omega t \cos \phi) \]
\[ + IX_C(- \sin(\omega t \sin \phi + \cos \omega t \cos \phi) \]

When time varies, the sin function may be 1 while the cos function may vanish and vice versa. The sum of coefficients of \( \sin(\omega t) \) and the sum of coefficients of \( \cos(\omega t) \) must separately vanish.
Solve for the unknown amplitude and phase of the current.

\textit{Equate terms proportional to \cos \omega t and \sin \omega t}

\begin{align*}
V_0 &= I(R \cos \phi + (X_L - X_C) \sin \phi) \\
0 &= R \sin \phi + (X_C - X_L) \cos \phi \\
\tan \phi &= (X_L - X_C)/R \\
I &= V_0/(R \cos \phi + (X_L - X_C) \sin \phi) \\
&= V_0/[R \cos \phi (1 + \tan^2 \phi)] = (V_0/R) \cos \phi \\
&= \frac{V_0}{R(1 + \tan^2 \phi)^{\frac{1}{2}}} = \frac{V_0}{\sqrt{R^2 + (X_C - X_L)^2}}
\end{align*}
Alternate solution using phasor diagram

- The individual phasor diagrams can be combined.
- Here a single phasor $I_{\text{max}}$ is used to represent the current in each element.
- From the phasor diagram, $\Delta V_{\text{max}}$ can be calculated in terms of $VR = IR$, $V_L = I \times X_L$, and $V_C = I \times X_C$.

\[
\Delta V_{\text{max}} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} \\
= \sqrt{(I_{\text{max}} R)^2 + (I_{\text{max}} X_L - I_{\text{max}} X_C)^2} \\
\Delta V_{\text{max}} = I_{\text{max}} \sqrt{R^2 + (X_L - X_C)^2}
\]
Impedance and resonance frequency

- The current in an RLC circuit can be expressed in terms of the impedance of the RLC combination called $Z$. Impedance has units of ohms.

- At high frequency, $\phi$ is positive $X_L > X_C$, and the current lags the applied voltage. The circuit is more inductive than capacitive.

- At low frequency, $\phi$ is negative, $X_L < X_C$, and the current leads the applied voltage. The circuit is more capacitive than inductive.

- If $\phi$ is zero, $X_L = X_C$. The impedance has a minimum and the current a maximum. The circuit is purely resistive. This occurs at the “resonance frequency” $(LC)^{-1/2}$.

\[
V_0 = IZ \\
Z = \sqrt{R^2 + (X_L - X_C)^2}
\]

\[
X_L = X_C \\
\omega L = \frac{1}{\omega C} \\
\omega = \frac{1}{\sqrt{LC}} \equiv \omega_R
\]
The average power delivered by the AC source is converted to internal energy in the resistor.

\[ P_{\text{avg}} = \frac{1}{2} I_{\text{max}} \Delta V_{\text{max}} \cos \phi = I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi \]

\[ \cos \phi \] is called the power factor of the circuit

We can also find the average power in terms of \( R \).

\[ P_{\text{avg}} = I_{\text{rms}}^2 R \]

When the load is purely resistive, \( \phi = 0 \) and \( \cos \phi = 1 \)

\[ P_{\text{avg}} = I_{\text{rms}} \Delta V_{\text{rms}} \]
Resonance occurs at the same frequency regardless of the value of R.

As R decreases, the curve becomes narrower and taller.

Theoretically, if R = 0 the current would be infinite at resonance. Real circuits always have some resistance.

The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor, Q.

\[ Q = \frac{\omega_o}{\Delta \omega} = \frac{(\omega_o L)}{R} \]

\( \omega_o \) is the width of the curve, measured between the two values of \( \omega \) for which \( P_{\text{avg}} \) has half its maximum value.

The current reaches its maximum value at the resonance frequency \( \omega_0 \).
An AC transformer consists of two coils of wire wound around a core of iron.

The side connected to the input AC voltage source is called the primary and has $N_1$ turns. The other side, called the secondary, is connected to a resistor and has $N_2$ turns.

The core is used to increase the magnetic flux and to provide a medium for the flux to pass from one coil to the other.

Assuming all the magnetic flux circulates through both coils, the voltages are related by the ratio of the number of turns.

The power input into the primary equals the power output at the secondary.

- $I_1 \Delta V_1 = I_2 \Delta V_2$

- The equivalent resistance of the load resistance when viewed from the primary is given by the square of the turns ratio.
Rectifier Circuit

- A diode has low resistance to current flow in one direction and high resistance to current flow in the opposite direction.

- With no capacitor, the alternating current in the load resistor is reduced (ideally to zero) in the positive portion of the cycle. (half wave rectification).

- The capacitor buffers the voltage - we have a simple AC-to-DC converter (with ripple).

- The transformer reduces the 120 V AC to the 6 V or 9 V typically needed.
LRC circuit demonstrations
A filter circuit is used to smooth out or eliminate a time-varying signal such as the ripple in the output of an AC-to-DC converter.

At low frequencies, the reactance and voltage across the capacitor are high.

As the frequency increases, the reactance and voltage decrease.
A high-pass filter is designed to preferentially pass signals of higher frequency and block lower frequency signals.

At low frequencies, the capacitor has high reactance and much of the applied voltage appears across the capacitor.

At high frequencies, the capacitive reactance is small and the voltage appears across the resistor.