Electrical appliances in the house use alternating current (AC) circuits.

If an AC source applies an alternating voltage of frequency $f$ to a series circuit containing resistor, inductor, and/or capacitor, the voltage across each element and the current through it will, once a steady state is established, alternate at frequency $f$.

We will determine the magnitude and time delay of each voltage and current.

Any applied voltage can be represented as combination of AC voltages of different frequencies. If we know the response for any frequency, we can construct the response to ANY applied voltage.
The output of an AC power source is sinusoidal and varies with time according to the following equation:

- $\Delta v = \Delta V_{\text{max}} \sin \omega t$

- $\Delta v$ is the instantaneous voltage.

- $\Delta V_{\text{max}}$ is the maximum output voltage of the source and is called the **voltage amplitude**.

- $\omega$ is the angular frequency of the AC voltage.

$$\omega = \frac{2\pi}{T} = 2\pi f$$
AC line voltage

Commercial electric power plants in the US use a frequency of 60 Hz.

What is the angular frequency? What is the period in seconds?
AC line frequency

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What is the angular frequency? What is the period in seconds?

- The frequency \( f = 60 \) Hz corresponds to an angular frequency of \( 2\pi f \) or 377 rad/s.

- The frequency \( f = 60 \) Hz = 60 s\(^{-1}\) corresponds to period \( T = 1/f = (1/60) \) s = 0.017 s.
The root mean square of an AC voltage is an alternate measure of its amplitude.

- The rms is $1/2^{1/2}$ or 0.707 of the peak positive value.

$$v_{rms} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 dt}$$

$$v(t) = V_0 \sin \omega t = V_0 \sin \frac{2\pi t}{T}$$

$$v_{rms}^2 = V_0^2 \frac{1}{T} \int_0^T \sin^2(\omega t)^2 dt$$

$$x = \omega t; \quad dx = \frac{2\pi}{T} dt$$

$$v_{rms}^2 = V_0^2 \frac{1}{2\pi} \int_0^{2\pi} \sin^2 x dx = \frac{V_0^2}{2}$$

$$v_{rms} = \frac{V_0}{\sqrt{2}}$$
In the US, the AC line voltage is said to be 120 volts. This is the root mean square value. What is the amplitude/maximal voltage?
AC line voltage

* In the US, the AC line voltage is said to be 120 volts. This is the root mean square value. What is the amplitude/maximal voltage?

* The maximum is $2^{1/2} \times 120 \, \text{V} = 170 \, \text{V}$.

* Note: The amplitude is NOT tightly controlled. Measurement results may vary by as much as 10%.

* All of Europe, Africa, Asia, Australia, New Zealand and most of South America use a supply that is within 6% of 230 V (rms), whereas Japan, Taiwan, North America and some parts of northern South America use a voltage between 100 and 127 V. The 230 V standard has become the most widespread.
Resistors in an AC Circuit

- Consider a circuit consisting of an AC source and a resistor.

- Neglect any response lag in a idealized resistor model and assume Ohm’s Law applies. The instantaneous voltage and current are related by the resistance $R$.

- $v(t) = i(t)R$

- The current and voltage are “in phase.” There is no time lag ideally. In reality, a resistor has a small inductance $L$ and there is a time lag $L/R$. 

\[
\Delta v = \Delta V_{\text{max}} \sin \omega t
\]
Average power

* What is the time averaged power dissipated in a 100 Ohm resistor subject to US line voltage?
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* What is the time averaged power dissipated in a 100 Ohm resistor subject to US line voltage?

* The instantaneous power is \( P(t) = i(t)v(t) = i(t)^2 R \). The rms voltage \( V_{\text{rms}} =120 \text{ V} \) and amplitude \( V_0 \) are known. The time average is

\[
<\!P(t)\!> = <\!i(t)v(t)\!> = <\!i(t)^2 \!> R = i_{\text{rms}}^2 R = V_{\text{rms}}^2 / R = RV_0^2 / 2
\]

* \( <\!P(t)\!> = V_{\text{rms}}^2 / R = (120 \text{ V})^2 / 100 \text{ Ohm} = 12^2 \text{ W} = 144 \text{ W} \)
Phasor Diagram

- To simplify the analysis of AC circuits, a phasor diagram can be used.

- A phasor is a vector whose length is proportional to the maximum value of the variable it represents.

- The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.

- The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

The current and the voltage phasors are in the same direction because the current is in phase with the voltage.
Recall the series LR circuit. If a constant voltage is suddenly applied, at first no current appears across the inductor - the back emf due to the quickly changing current compensates the driving voltage.

Ultimately, steady current $i = \frac{V}{R}$ is achieved and the inductor behaves like a conducting wire, its inductive property irrelevant.

The characteristic time is $\frac{L}{R}$. 

\[
\begin{align*}
i(t) &= I_{\text{max}} \left( 1 - e^{-\frac{t}{\frac{L}{R}}} \right) \\
i(t) &= \frac{E}{R} \left( 1 - e^{-\frac{t}{\frac{L}{R}}} \right)
\end{align*}
\]
Consider an AC voltage source applied to an ideal inductor.

Kirchhoff’s loop rule can be applied and gives the instantaneous current.

This shows that the instantaneous current $i_L$ in the inductor and the instantaneous voltage $\Delta v_L$ across the inductor are out of phase by $(\pi/2)$ rad = $90^\circ$.

\[
\Delta v_L(t) = \Delta v(t)
\]

\[
L \frac{di}{dt} = \Delta V_{max} \sin \omega t
\]

\[
i_L(t) = -\frac{\Delta V_{max}}{\omega L} \cos \omega t
\]

\[
= \frac{\Delta V_{max}}{\omega L} \sin(\omega t - \frac{\pi}{2})
\]
The voltage across the inductor is zero when the current is at a maximum and therefore not (momentarily) changing, i.e. when there is no back emf.

For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by $90^\circ (\pi/2)$.
The phasors representing the applied voltage and the corresponding current rotate but are always at 90° with respect to each other.

This represents the phase difference between the current and voltage.

Specifically, the current lags behind the voltage by 90°.

The importance of this will be clear when we add phasors as vectors to represent circuit element combinations.
The magnitude/amplitude of the current is proportional to the magnitude/amplitude of the drive voltage.

We write this proportionality in analogy to Ohm’s Law $V = RI$ as $V_{\text{max}} = X_L i_{\text{max}}$ where $X_L$ is called the **inductive reactance** and has units of Ohms.

At low frequency, the reactance is small and a large current appears. The inductor behaves like a conducting short.

At high frequency, the back emf chokes off the current. The reactance is large and current small.

\[
\begin{align*}
\Delta v(t) &= \Delta V_{\text{max}} \sin \omega t \\
i_L(t) &= -\frac{\Delta V_{\text{max}}}{\omega L} \cos \omega t \\
&= \frac{\Delta V_{\text{max}}}{\omega L} \sin(\omega t - \frac{\pi}{2}) \\
\Delta V_{\text{max}} &= X_L \Delta i_{\text{max}} \\
X_L &= \omega L
\end{align*}
\]
Inductive reactance

- A coil of copper wire is connected between the terminals of a variable-frequency fixed-amplitude AC voltage source.

- Is there more current through the wire at low frequency or at high frequency?
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\[ X_L = \omega L \]

\[ \Delta v = \Delta V_{\text{max}} \sin \omega t \]