Charge and mass of the electron

- Thomson’s e/m experiment
- Electrons are accelerated from the cathode. They are deflected by electric and magnetic fields. The beam of electrons strikes a fluorescent screen.
- e/m was measured.
- The electric field required to levitate singly charged oil drops was used by Millikan to determine e. Together, m is determined.

General motion in a uniform magnetic field

- The magnetic force has no component along the direction of the magnetic field B.
- The component of velocity along the field is constant so the general motion is a helix - uniform motion along B and uniform circular motion about B.
- The angular frequency \( \omega = \frac{qB}{m} \) of revolution is called the cyclotron frequency.

Combined E and B fields

- Motion in combined electric and magnetic fields is governed the total force
- \( \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \)
- If \( \mathbf{E} \) and \( \mathbf{B} \) are uniform and constant and at right angles as shown, the total force vanishes for \( v = \frac{E}{B} \). Only particles with this speed pass straight through such a “velocity selector.”

Mass spectrometer

- A mass spectrometer separates ions according to their mass-to-charge ratio.
- In one design, a beam of ions passes through a velocity selector and enters a second magnetic field.
- After entering the second magnetic field, the ions move in a semicircle of radius \( r \) before striking a detector at \( P \).
- If the ions are positively charged, they deflect to the left.
- If the ions are negatively charged, they deflect to the right.
**Sector mass spectrometer**

- In a simple spectrometer, ions are created in a gas plasma and accelerated. The deflection by the magnetic field for fixed energy depends on q/m.
- Different isotopes (nuclear masses) are separated.
- More sophisticated high resolution spectrometers use high speed beams.

**Cyclotron**

- In a cyclotron, an alternating voltage is applied between two cup-shaped “dees” accelerating ions in the gap. In the interior of the “dees,” the electric field vanishes. The ions travel in a circular orbit in a uniform constant magnetic field.
- The voltage alternates at the cyclotron frequency so that with each gap crossing, the energy is increased.

**Sector focused cyclotron**

- Advanced cyclotrons use multiple magnets and multiple acceleration gaps to achieve improved beam stability and higher energy (520 MeV at TRIUMPH).

**Large hadron collider**

- Two proton beams counter circulate and collide. During acceleration, protons are given repeated kicks at a fixed location while B-field increases to keep beams in the tunnel.
- \( r = 4.3 \text{ km}, B(\text{max}) = 8 \text{ T}, E = 7 \text{ TeV} = 7,000,000,000,000 \text{ Volts equivalent} \)
Magnetic force on a wire

- When an electric current flows in a conducting wire in a magnetic field, the magnetic force acts on the electrons.
- The electrons are bound electrically to the conductor so the force is transferred to the wire.
- Hence, a current carrying wire is subject to a magnetic force.

Force on a wire

- The total force is the force on one charge times the number of charges.
- For electron drift speed
  \[ \mathbf{v}_d = \mathbf{v}_d \]
- and density \( n \), the total force on a length \( L \) of wire of cross sectional area \( A \) is
  \[ \mathbf{F} = nLA(-e)\mathbf{v}_d \times \mathbf{B} = ILn\mathbf{B} \]

Generalization

- The magnetic force on curved wires (circuits) is constructed by considering small elements of the circuit.
- For a vector element of length \( ds \), \( d\mathbf{F} = I ds \times \mathbf{B} \).
- The total force on the current carrying wire is obtained by integrating over elements of the wire.

\[
d\mathbf{F}_B = I d\mathbf{s} \times \mathbf{B}
\]

\[
\mathbf{F}_B = I \int_{a}^{b} d\mathbf{s} \times \mathbf{B}
\]

Torque on a current loop

- Consider a rectangular circuit in a magnetic field as shown.
- There is a force only on sides 2 & 4 since they are not parallel to the field:
  \[ \mathbf{F}_2 = \mathbf{F}_4 = Ia\mathbf{B} \]
- The direction of \( \mathbf{F}_2 \) is out of the page. The direction of \( \mathbf{F}_4 \) is into the page. The forces are equal and in opposite directions, but not along the same line of action.
- The forces produce a torque around point \( O \).
Torque on a current loop

- Assume the magnetic field makes an angle of $\theta < 90^\circ$ with a line perpendicular to the plane of the loop.
- The net torque about point O will be $\tau = IAB \sin \theta$.
- The vector torque may be written in terms of a vector area $A$ perpendicular to the loop of magnitude equal to the area and direction given by the right hand rule.

\[ \tau = I \vec{A} \times \vec{B} \]

Magnetic dipole moment

- The product $m = IA$ is defined as the magnetic dipole moment of the loop, often called the simply the magnetic moment and denoted by Greek “mu”.
- SI units: A $\cdot$ m$^2$
- Torque $N$ (or Greek “tau”) in terms of magnetic moment:
  \[ N = m \times B \]
- Valid for a loop of any shape.

Magnetic energy

- The magnetic moment of this loop is up.
- The torque is trying to align the moment with the field.
- The magnetic potential energy is
  \[ U = -m \times B \]

Electric motor

- Magnetic torque on a steady current loop tries to align the magnetic moment with the field.
- If the current or field is reversed synchronously a push-pull becomes a push-pull resulting in uni-polar angular acceleration!
Hall effect

- When a current carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field.

- This phenomena is known as the Hall effect.

- It arises from the deflection of charge carriers to one side of the conductor as a result of the magnetic forces they experience.

- The Hall effect gives information regarding the sign of the charge carriers and their density.

- It can also be used to measure magnetic fields.