Lecture 26

Goals:

- Understand pressure in liquids
  - Use Archimedes’ principle to understand buoyancy
- Understand the equation of continuity
- Use an ideal-fluid model to study fluid flow.

Fluids

- At ordinary temperatures, matter exists in one of three states
  - Solid: Has a shape and forms a surface
  - Liquid: Has no shape but forms a surface
  - Gas: No shape and does not form a surface

Increasing temperature

Fluids: Liquids or Gases

- Shape determined by container
- No macroscopic gaps
- Forces exerted by fluids are always normal to the surface.
- Fluids can flow

Fluids:

- They have mass
- They have weight
- They carry energy
- They have velocity when flow
- Newton’s Laws apply

Fluids

An intrinsic parameter of a fluid

Density

\[ \rho = \frac{m}{V} \]

units:

\[ \text{kg/m}^3 = 10^{-3} \text{ g/cm}^3 \]

\[ \rho_{(\text{water})} = 1.000 \times 10^3 \text{ kg/m}^3 = 1.000 \text{ g/cm}^3 \]

\[ \rho_{(\text{ice})} = 0.917 \times 10^3 \text{ kg/m}^3 = 0.917 \text{ g/cm}^3 \]

\[ \rho_{(\text{air})} = 1.29 \text{ kg/m}^3 = 1.29 \times 10^{-3} \text{ g/cm}^3 \]

\[ \rho_{(\text{Hg})} = 13.6 \times 10^3 \text{ kg/m}^3 = 13.6 \text{ g/cm}^3 \]

\[ \rho_{(W \text{ or Au})} = 19.3 \times 10^3 \text{ kg/m}^3 = 19.3 \text{ g/cm}^3 \]

What is the SI unit of pressure?

A. Pascal
B. Atmosphere
C. Bernoulli
D. Young
E. p.s.i.

Units:

1 N/m² = 1 Pa (Pascal)  
1 bar = 10⁵ Pa  
1 mbar = 10² Pa  
1 torr = 133.3 Pa

1 atm = 1.013 x 10⁵ Pa  
1 bar = 10¹³ mbar  
1 mbar = 760 Torr  
1 atm = 14.7 lb/ in² (=PSI)
A bed of nails

1. Distributing the force over a large area ...

Ping pong ball bazooka

1. What is the initial force on the ping pong ball?
2. Assuming this force is constant, how fast is the ping pong ball travelling when it leaves the tube?

\[ F = PA = 1\text{atm} \times \pi r^2 = 10^3 \times \pi (0.020)^2 \text{ N} \]

\[ F = 130 \text{ N} \]

\[ W = Fd = 130 \text{ N} \times 2 \text{ m} = 260 \text{ J} \]

\[ W = \frac{1}{2}mv^2 = 260 \text{ J} \quad v = 440 \text{ m/s} \]

This is a crude model...the speed of sound in air is 330 m/s

Pressure vs. Depth

Incompressible Fluids (liquids)

1. When the pressure is small, relative to the bulk modulus of the fluid, we can treat the density as constant independent of pressure: incompressible fluid
2. For an incompressible fluid, the density is the same everywhere, but the pressure is NOT!
3. \( P(y) = P_0 - y \frac{\rho}{g} \)
4. Gauge pressure (subtract \( P_0 \))
5. \( P_{\text{Gauge}} = P(y) - P_0 \)

\[ P_2 = P_1 - \rho gy \]

Pressure vs. Depth in water

\[ \rho_{\text{water}} = 1.0 \text{ gm/cm}^3 = 10^3 \text{ kg/m}^3 \]

\[ g = 10 \text{ m/s}^2 \]

\[ y = -10 \text{ m} \]

\[ P_2 = P_i + 10^5 \text{ N/m}^2 \]

Every 10 meters the pressure increases by 1 atm

Pascal’s Principle

1. For a uniform fluid in an open container pressure same at a given depth independent of the container

2. Fluid level is the same everywhere in a connected container, assuming no surface forces

Exercise

1. Does \( P_A = P_B \)?
Exercise
Pressure

What happens with two fluids??

Consider a U tube containing liquids of density $\rho_1$ and $\rho_2$ as shown:

Compare the densities of the liquids:

(A) $\rho_1 < \rho_2$  (B) $\rho_1 = \rho_2$  (C) $\rho_1 > \rho_2$

Pressure Measurements: Barometer

Invented by Torricelli

A long closed tube is filled with mercury and inverted in a dish of mercury

The closed end is nearly a vacuum

Measures atmospheric pressure as $1 \text{ atm} = 0.760 \text{ m (of Hg)}$

Pascal’s Principle in action:
Hydraulics, a force amplifier

Consider the system shown:

This force is transmitted through the liquid to create an upward force $F_2$.

Pascal’s Principle says that increased pressure from $F_1 (F_1 / A_1)$ is transmitted throughout the liquid.

$$ F_2 = F_1 \frac{A_2}{A_1} $$

Pascal’s Principle: Example 2

Now consider the set up shown on right.

Mass $M$ is placed on the larger bore piston, $A_{10}$.

If $A_2 = 2A_1$, how do $d_a$ and $d_b$ compare?

This results in a difference $d$ in the liquid levels on the left.

Equilibrium occurs when the pressures at P (left & right) are equal:

$$ P_1 = P_2 $$

$$ F_1 / A_1 = F_2 / A_2 $$

$$ P g A d / A_1 = P g A d / A_2 $$

$$ d_a = d_b $$

Archimedes’ Principle: A Eureka Moment

Suppose we weigh an object in air (1) and in water (2).

How do these weights compare?

$W_1 < W_2$  $W_1 = W_2$  $W_1 > W_2$

Buoyant force = Weight of the fluid displaced

The Golden Crown

In the first century BC the Roman architect Vitruvius related a story of how Archimedes uncovered a fraud in the manufacture of a golden crown commissioned by Hiero II, the king of Syracuse. The crown (corona in Vitruvius’s Latin) would have been in the form of a wreath, such as one of the three pictured from grave sites in Macedonia and the Dardanelles. Hiero would have placed such a wreath on the statue of a god or goddess. Suspecting that the goldsmith might have replaced some of the gold given to him by an equal weight of silver, Hiero asked Archimedes to determine whether the wreath was pure gold. And because the wreath was a holy object dedicated to the gods, he could not disturb the wreath in any way. (In modern terms, he was to perform nondestructive testing). Archimedes’ solution to the problem, as described by Vitruvius, is neatly summarized in the following excerpt from an advertisement:

The solution which occurred when he stepped into his bath and caused it to overflow was to put a weight of gold equal to the crown, and known to be pure, into a bowl which was filled with water to the brim. Then the gold would be removed and the king’s crown put in, in its place. An alloy of lighter silver would increase the bulk of the crown and cause the bowl to overflow.

Sink or Float?

1. The buoyant force is equal to the weight of the liquid that is displaced.
2. If the buoyant force is larger than the weight of the object, it will float; otherwise it will sink.
3. We can calculate how much of a floating object will be submerged in the liquid:
   - Object is in equilibrium: \( F_B = mg \)
   - \( \rho_{\text{liquid}} \cdot g \cdot V_{\text{liquid}} = \rho_{\text{object}} \cdot g \cdot V_{\text{object}} \)
   - \( \frac{V_{\text{liquid}}}{V_{\text{object}}} = \frac{\rho_{\text{object}}}{\rho_{\text{liquid}}} \)

Bar Trick

What happens to the water level when the ice melts?

Expt. 1
- A. It rises
- B. It stays the same
- C. It drops

Expt. 2
- Bar Trick

Exercise

\( V_1 = V_2 = V_3 = V_4 = V_5 \)
\( m_1 < m_2 < m_3 < m_4 < m_5 \)
What is the final position of each block?

Exercice

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\( m_1 < m_2 < m_3 < m_4 < m_5 \)
What is the final position of each block?

Home Exercise Buoyancy

1. A small lead weight is fastened to a large styrofoam block and the combination floats on water with the water level with the top of the styrofoam block as shown.
2. If you turn the styrofoam + Pb upside-down, What happens?
   - (A) It sinks
   - (B) Styrofoam
   - (C) Styrofoam
   - (D) Styrofoam

Exercise Buoyancy

1. A small lead weight is fastened to a large styrofoam block and the combination floats on water with the water level with the top of the styrofoam block as shown.
2. If you turn the styrofoam + Pb upside-down, What happens (assuming density of Pb > water)?
   - (A) It sinks
   - (B) Styrofoam
   - (C) Styrofoam
   - (D) Styrofoam
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Home Exercise

More Buoyancy

Two identical cups are filled to the same level with water. One of the two cups has plastic balls floating in it.

Which cup weighs more?

(A) Cup I (B) Cup II (C) the same (D) can’t tell

Exercise

More Buoyancy

Two identical cups are filled to the same level with water. One of the two cups has plastic balls floating in it.

Which cup weighs more?

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Home Exercise

Even More Buoyancy

A plastic ball floats in a cup of water with half of its volume submerged. Next some oil ($\rho_{\text{oil}} < \rho_{\text{ball}} < \rho_{\text{water}}$) is slowly added to the container until it just covers the ball.

Relative to the water level, the ball will:

Hint 1: What is the buoyant force of the part in the oil as compared to the air?

(A) move up (B) move down (C) stay in same place

For Tuesday

All of chapter 14