Physics 201 – Lecture 22

Lecture 21

Goals:

• Use Free Body Diagrams prior to problem solving
• Introduce Young’s, Shear and Bulk modulus

Exam 3: Wednesday, April, 18th 7:15-8:45 PM

Statics

Equilibrium is established when

Translational motion \[ \sum \vec{F} = 0 \]

Rotational motion \[ \sum \vec{\tau} = 0 \]

In 3D this implies SIX expressions (x, y & z)

Balancing Acts

So, where is the center of mass for this construction?

Between the arrows!

Example problem

1. What are the minimum and maximum masses of the bottle that will allow this system to remain in balance?

2. X center of mass,
   \[ x = -0.1 \text{ m} < x < 0.1 \text{ m} \]
   \[ x_{CM} = \frac{\sum x_{CM}m_i}{\sum m_i} \]
   \[ x_{CM} = \frac{-0.5(2) + 0.4(M)}{M + 2} = 1 \]
   \[ \frac{-10 + 4M}{M + 2} = 1 \]
   \[ M = 4 \text{ kg} \]

Example problem

1. Check torque condition \((g = 10 \text{ m/s}^2)\)
2. \[ x = -0.1 \text{ m} < x < 0.1 \text{ m} \]

\[ \sum \tau_z = -40(0.3) + 20(0.6) \]

\[ \sum \tau_z = (-12 + 12) \text{ Nm} \]

\[ \sigma \tau_z = 0 \text{ Nm} \]

\[ M_{\text{max}} = 40 \text{ kg} \]

\[ M_{\text{min}} = 16 \text{ kg} \]
Real Physical Objects

- Representations of matter
  - Particles: No size, no shape, can only translate along a path
  - Extended objects are collections of point-like particles
    They have size and shape, so, to better reflect their motion, we introduce the center of mass

Rigid objects: Translation + Rotation
Deformable objects:
  - Regular solids: Shape/size changes under stress (applied forces)
    Reversible and irreversible deformation
  - Liquids: They do not have fixed shape
    Size (aka volume) will change under stress

Changes in length: Young’s modulus

- Young’s modulus: measures the resistance of a solid to a change in its length.

\[ Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_0} \]

Real Materials have a complex behavior

\[ \frac{AY}{L} \Delta L = F \]
\[ k \Delta L = F \]

Hooke’s Law

Changes in volume: Bulk Modulus

- Bulk modulus: measures the resistance of solids or liquids to changes in their volume.

\[ B = \frac{F/A}{\Delta V/V_i} \]

Changes in shape: Shear Modulus

Applying a force perpendicular to a surface

\[ S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h} \]

Example values

<table>
<thead>
<tr>
<th>Substance</th>
<th>Young’s Modulus (N/m²)</th>
<th>Shear Modulus (N/m²)</th>
<th>Bulk Modulus (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tungsten</td>
<td>35 x 10¹⁰</td>
<td>14 x 10¹⁰</td>
<td>20 x 10¹⁰</td>
</tr>
<tr>
<td>Steel</td>
<td>20 x 10¹⁰</td>
<td>8.4 x 10¹⁰</td>
<td>6 x 10¹⁰</td>
</tr>
<tr>
<td>Copper</td>
<td>11 x 10¹⁰</td>
<td>4.2 x 10¹⁰</td>
<td>14 x 10¹⁰</td>
</tr>
<tr>
<td>Brass</td>
<td>9.1 x 10¹⁰</td>
<td>3.5 x 10¹⁰</td>
<td>6.1 x 10¹⁰</td>
</tr>
<tr>
<td>Aluminium</td>
<td>7.0 x 10¹⁰</td>
<td>2.5 x 10¹⁰</td>
<td>7.0 x 10¹⁰</td>
</tr>
<tr>
<td>Glass</td>
<td>6.5–7.8 x 10¹⁰</td>
<td>2.6–3.2 x 10¹⁰</td>
<td>5.0–5.5 x 10¹⁰</td>
</tr>
<tr>
<td>Quartz</td>
<td>5.6 x 10¹⁰</td>
<td>2.6 x 10¹⁰</td>
<td>2.7 x 10¹⁰</td>
</tr>
<tr>
<td>Water</td>
<td>—</td>
<td>—</td>
<td>8.21 x 10¹⁰</td>
</tr>
<tr>
<td>Mercury</td>
<td>—</td>
<td>—</td>
<td>2.8 x 10¹⁰</td>
</tr>
</tbody>
</table>

Carbon nanotube 100 x 10¹⁰
Example: A flying swing

A flying swing, as shown, consists of a 5.0 m horizontal ball and, at its end, a hanging wire 5.0 m long. The wire has a cross sectional area of 1.0x10^4 m^2. The swing turns so that the wire makes a 45° angle with respect to the vertical. Assuming g=10 m/s^2 and the wire’s Young’s modulus is 20x10^10 N/m^2 (steel), how much will the wire stretch?

\[ R = (5.0 + 5.0 \sin 45^\circ) \text{m} = 8.53 \text{m} \]
\[ \Sigma F_y = 0 = -500N + T_y \]
\[ \Sigma F_x = -m \ddot{a} R = -T_x \]
\[ T_y = T_x = T \frac{\sin 45^\circ}{\cos 45^\circ} \]
\[ T = 707 \text{N} \]

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To the bottom of the ocean

What is the density change at the bottom of the ocean for liquid mercury?
Facts: 1 atm = 1.013 x 10^5 N/m^2
So 1 atm increase for every 10 m.
Thus, 10 km is 10000 m or 1000 atm & B = 3 x 10^10 N/m^2 for Hg

\[ B = \frac{F/A}{\Delta V/V_i} \]
\[ \frac{\Delta V}{V_i} = \frac{F/A}{B} \]
\[ \Delta V = \frac{10^9}{3 \times 10^{10}} \text{N/m}^2 = 0.33 \times 10^{-2} \]

The ice bomb

How does this compare to the case when water freezes?
On freezing the fractional change in volume is about 0.08.
B = 0.2 x 10^10 N/m^2 for ice (close to that of water)

\[ \frac{\Delta V}{V_i} = \frac{P}{2 \times 10^{10}} \text{N/m}^2 = 8 \times 10^{-2} \]
\[ P = 1.6 \times 10^7 \text{ N/m}^2 \]
\[ P = 1.6 \times 10^3 \text{ atm} \]

For Tuesday

Review