Lecture 19

Goals:

• Specify rolling motion (center of mass velocity to angular velocity)
• Compare kinetic and rotational energies with rolling
• Work combined force and torque problems
• Revisit vector cross product
• Introduce angular momentum

Example: Rotating Rod

A uniform rod of length L=0.5 m and mass m=1 kg is free to rotate on a frictionless pin passing through one end as shown. The rod is released from rest in the horizontal position. What is:

(A) its angular speed when it reaches the lowest point?
(B) its initial angular acceleration?
(C) initial linear acceleration of its free end?

Connection with Center-of-mass motion

1. If an object of mass M is moving linearly at velocity \( V_{CM} \) without rotating then its kinetic energy is

\[
K_T = \frac{1}{2} MV_{CM}^2
\]

2. If an object with moment of inertia \( I_{CM} \) is rotating in place about its center of mass at angular velocity \( \omega \) then its kinetic energy is

\[
K_T = \frac{1}{2} I_{CM} \omega^2
\]

3. If the object is both moving linearly and rotating then

\[
K = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} MV_{CM}^2
\]
But what of Rolling Motion

1. Consider a cylinder rolling at a constant speed.
2. Contact point has zero velocity in the laboratory frame.
3. Center of wheel has a velocity of $V_{CM}$
4. Top of wheel has a velocity of $2V_{CM}$

Rolling Motion

Now consider a cylinder rolling at a constant speed.

$V_{CM}$

The cylinder is rotating about CM and its CM is moving at constant speed ($V_{CM}$). Thus its total kinetic energy is given by:

$$K_{TOT} = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} M V_{CM}^2$$

Concept question

1. For a hoop of mass M and radius R that is rolling without slipping, which is greater, its translational or its rotational kinetic energy?
   
   A. Translational energy is greater
   B. Rotational energy is greater
   C. They are equal
   D. The answer depends on the radius
   E. The answer depends on the mass

Example: Rolling Motion

1. A solid cylinder is about to roll down an inclined plane.
   What is its speed at the bottom of the plane?
2. Use Work-Energy theorem

$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I_{CM} \omega^2$$

and $v = \omega R$

$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} (\frac{1}{2} M R^2)(v/R)^2 = \frac{3}{4} M v^2$$

$$v = 2(gh/3)^{1/2}$$

Question: The Loop-the-Loop … with rolling

1. To complete the loop-the-loop, how does the height $h$ compare with rolling as to that with frictionless sliding?
   
   A. $h_{rolling} > h_{sliding}$
   B. $h_{rolling} = h_{sliding}$
   C. $h_{rolling} < h_{sliding}$

Example: Loop-the-Loop with rolling

1. To complete the loop the loop, how high do we have to release a ball with radius \( r \) (\( r \ll R \))?  
2. Condition for completing the loop the loop: Circular motion at the top of the loop (\( a_c = v^2 / R \))  
3. Use fact that \( E = U + K = \text{constant} \) (or work energy)  

\[ U = mgh \]
\[ U = mg2R \]

Recall that \( g \) is the source of the centripetal acceleration and \( N \) just goes to zero is the limiting case. Also recall the minimum speed at the top is \( \frac{gR}{r} \).

\[ v_{\text{Tangential}} = \sqrt{gR} \]

Example: Loop-the-Loop ... Energy Conservation

1. If rolling then ball has both rotational and CM motion!  
2. \( E = U + K_{\text{CM}} + K_{\text{Rot}} = \text{constant} = mgh \) (at top)  
3. \( E = mg2R + \frac{1}{2} m v^2 = mg \)  

\[ h = \frac{5}{2} R + \frac{1}{5} R \]

Just a little bit more....

Torque : Rolling Motion (center of mass point)

1. A solid cylinder, with mass \( m \) and radius \( R \), is rolling without slipping down an inclined plane.  
2. What is its angular acceleration \( \alpha \)?  
3. \( \Sigma F = ma_c = mg \sin \theta - f_s \)
   \[ mRa_c = mgR \sin \theta - Rf_s \]

\[ \alpha = -mgR \sin \theta / (mR^2 + I_{cm}) = -2g \sin \theta / 3R \]

Torque : Rolling Motion (contact point)

1. A solid cylinder, with mass \( m \) and radius \( R \), is rolling without slipping down an inclined plane.  
2. What is its angular acceleration \( \alpha \)?  
3. \( \Sigma \tau = -R mg \sin \theta = I \alpha \)
   \[ I = mR^2 + I_{cm} \]

\[ \alpha = -mgR \sin \theta / (mR^2 + I_{cm}) \]
\[ \alpha = 2g \sin \theta / 3R \]

Torque : Come back spool (contact point)

1. A solid cylinder, with mass \( m \), inner radius \( r \) and outer radius \( R \), being pulled by a horizontal force \( F \) at radius \( r \). If the cylinder does not slip then what is the angular acceleration?  
2. \( \Sigma \tau = -Rf = I \alpha \)
   \[ I = mR^2 + I_{cm} = 3/2 mR^2 \]

\[ \alpha = -Rf / (mR^2 + I_{cm}) = -2Fr / 3R \]

\[ \alpha = 2Fr / 3R \]
Torque: Limit of Rolling (center of mass)

1. A solid cylinder, with mass $m$ and radius $R$, being pulled by a horizontal force $F$ on its axis of rotation. If $\mu_s$ is the coefficient of static friction at the contact point, what is the maximum force that can be applied before slipping occurs?

2. $\Sigma F_y = 0 = N - mg$
3. $\Sigma \tau = -R f_s = l_{cm} \alpha$
4. $f_s \leq \mu_s N = \mu_s mg$ (maximum)
5. $a_{cm} = \frac{F - f_s}{l_{cm}}$

Modified Atwood’s Machine (More torques)

1. Two blocks, as shown, are attached by a massless string which passes over a pulley with radius $R$ and rotational inertia $I = \frac{1}{2} MR^2$. The string moves past the pulley without slipping. The surface of the table is frictionless.

2. What are the tensions in the strings?

3. $\Sigma F_y = m_1 a_y = -m_1 g + T_1$
4. $\Sigma F_x = m_2 a_x = -T$
5. $\Sigma \tau = I \alpha = R T_1 - R T$
6. Solve for $a$

Angular Momentum

1. We have shown that for a system of particles, the angular momentum $\mathbf{L}$ is conserved if $\Sigma \mathbf{F}_{\text{Ext}} = \frac{d\mathbf{L}}{dt} = 0$.

2. Are the rotational equivalents? $\vec{\tau} = \vec{r} \times \vec{F} = I \vec{\alpha}$

For Thursday

1. Read all of chapter 10 (gyroscopes will not be tested)
2. HW9