Lecture 10

Goals
- Describe Friction
- Problem solving with Newton’s 1st, 2nd and 3rd Laws
- Forces in circular and curvilinear motion

Another experiment: A modified Atwood’s machine
Two blocks, \( m_1 \) & \( m_2 \), are connected by a massless frictionless string/pulley on the table as shown. The table surface is frictionless and little \( g \) acts downward.

What is the acceleration of the horizontal block?
Requires two FBDs and Newton’s 3rd Law.

Mass 1
\[
\sum F_y = m_1 a_1y = T - m_1 g
\]

Mass 2
\[
\sum F_x = m_2 a_2x = -T
\]
\[
\sum F_y = 0 = N - m_2 g
\]

Correlated motion:
\[|a_{1y}| = |a_{2x}| \equiv a\]
If \( m_1 \) moves up moves \( m_2 \) right

A “special” contact force: Friction
- What does it do?
  - It opposes motion (velocity, actual or that which would occur if friction were absent!)
- How do we characterize this in terms we have learned?
  - Friction results in a force in a direction opposite to the direction of motion (actual or, if static, then “inferred”):

Friction...
- Friction is caused by the “microscopic” interactions between the two surfaces:
Friction: Static friction

Static equilibrium: A block with a horizontal force $F$ applied,

\[
\Sigma F_x = 0 = -F + f_s \\
\Sigma F_y = 0 = -N + mg \\
N = mg
\]

Direction: A force vector $\perp$ to the normal force vector $N$ and the vector is opposite to the direction of acceleration if $\mu$ were 0.

Magnitude: $f_s$ is proportional to the magnitude of $N$

As $F$ increases so does $f_s$

Static friction, at maximum (just before slipping)

Equilibrium: A block, mass $m$, with a horizontal force $F$ applied,

\[
\Sigma F_x = 0 = -F + f_s \\
\Sigma F_y = 0 = -N + mg \\
N = mg
\]

Direction: A force vector $\perp$ to the normal force vector $N$ and the vector is opposite to the direction of acceleration if $\mu$ were 0.

Magnitude: $f_s$ is proportional to the magnitude of $N$

As $F$ increases so does $f_s$

Kinetic or Sliding friction $(f_k < f_s)$

Dynamic equilibrium, moving but acceleration is still zero

\[
\Sigma F_x = 0 = -F + f_k \\
\Sigma F_y = 0 = -N + mg \\
N = mg
\]

As $F$ increases, $f_k$ remains nearly constant (but now there acceleration is acceleration)

Model of Static Friction (simple case)

Magnitude:

$f$ is proportional to the applied forces such that

\[ f_s \leq \mu_s N \]

$\mu_s$ called the "coefficient of static friction"

Direction:

Opposite to the direction of system acceleration if $\mu$ were 0

Sliding Friction

1. Direction: A force vector $\perp$ to the normal force vector $N$ and the vector is opposite to the velocity.

2. Magnitude: $f_k$ is proportional to the magnitude of $N$

\[ f_k = \mu_k N \quad (= \mu_k mg \text{ in the previous example}) \]

3. The constant $\mu_k$ is called the "coefficient of kinetic friction"

4. Logic dictates that $\mu_s > \mu_k$ for any system

Coefficients of Friction

<table>
<thead>
<tr>
<th>Material on Material</th>
<th>$\mu_s$ = static friction</th>
<th>$\mu_k$ = kinetic friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>steel / steel</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>add grease to steel</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>metal / ice</td>
<td>0.022</td>
<td>0.02</td>
</tr>
<tr>
<td>brake lining / iron</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>tire / dry pavement</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>tire / wet pavement</td>
<td>0.8</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Forces at different angles

Case 1: Downward angled force with friction
Case 2: Upwards angled force with friction
Cases 3, 4: Up against the wall

Questions:
- Does it slide?
- What happens to the normal force?
- What happens to the frictional force?

Many Forces are Conditional

- Notice what happens if we change the direction of the applied force
- The normal force can increase or decrease

An experiment

Two blocks are connected on the table as shown. The table has unknown static and kinetic friction coefficients.

**Design an experiment to find \( \mu_s \)**

\[
\begin{align*}
\Sigma F_x &= 0 = -T + \text{m}_1 g \\
\Sigma F_y &= 0 = T - \text{m}_2 g \\
T &= \text{m}_1 g = \text{m}_3 \text{m}_2 g \\
\mu_s &= \frac{\text{m}_1}{\text{m}_2}
\end{align*}
\]

A conceptual question: A flying bird in a cage

- You have a bird in a cage that is resting on your upward turned palm. The cage is completely sealed to the outside (at least while we run the experiment!). The bird is initially sitting at rest on the perch. It decides it needs a bit of exercise and starts to fly.

Question: How does the weight of the cage plus bird vary when the bird is flying up, when the bird is flying sideways, when the bird is flying down?

- Follow up question:
  
  So, what is holding the airplane up in the sky?

3rd Law: Static Friction with a bicycle wheel

- You are pedaling hard and the bicycle is speeding up. What is the direction of the frictional force?
- You are breaking and the bicycle is slowing down. What is the direction of the frictional force?

Static Friction with a bicycle wheel

- You are pedaling hard and the bicycle is speeding up. What is the direction of the frictional force?

Hint...you are accelerating to the right

- What is the direction of the frictional force?

\[ \mathbf{a} = \frac{F}{m} \]

\[ F_{\text{friction, on B from E}} \text{ is to the right} \]

\[ F_{\text{friction, on E from B}} \text{ is to the left} \]
**Home Exercise**

Newton’s Third Law

A fly is deformed by hitting the windshield of a speeding bus.

The force exerted by the bus on the fly is,

- A. greater than
- B. equal to
- C. less than

that exerted by the fly on the bus.

**Home Exercise**

Newton’s Third Law

A fly is deformed by hitting the windshield of a speeding bus.

The force exerted by the bus on the fly is,

- B. equal to

that exerted by the fly on the bus.

**Home Exercise**

Newton’s 3rd Law

Same scenario but now we examine the accelerations

A fly is deformed by hitting the windshield of a speeding bus.

The magnitude of the acceleration, due to this collision, of

- A. greater than
- B. equal to
- C. less than

that of the fly.

**Home Exercise**

Newton’s 3rd Law

Solution

By Newton’s third law these two forces form an interaction pair which are equal (but in opposing directions).

Thus the forces are the same

However, by Newton’s second law \( F_{\text{net}} = ma \) or \( a = \frac{F_{\text{net}}}{m} \).

So \( F_{b, f} = F_{f, b} = F_0 \) but \( |a_{\text{bus}}| = \frac{|F_0|}{m_{\text{bus}}} \ll |a_f| = \frac{|F_0|}{m_{\text{fly}}} \)

Answer for acceleration is (C)

**Force pairs on an Inclined plane**

Forces on the block (static case)

Forces on the plane by block

**The inclined plane coming and going (not static):**

The component of mg along the surface > kinetic friction

\[ \Sigma F_x = ma_x = mg \sin \theta \pm u_k N \]
\[ \Sigma F_y = ma_y = 0 = -mg \cos \theta + N \]

Putting it all together gives two different accelerations,

\( a_x = g \sin \theta \pm u_k g \cos \theta \). A tidy result but ultimately it is the process of applying Newton’s Laws that is key.
Inclined plane with “Normal” and Frictional Forces

1. Static Equilibrium Case
2. Dynamic Equilibrium (see 1)
3. Dynamic case with non-zero acceleration

Block weight is $mg$

Normal Force

$F_x = 0 = mg \sin \theta - f$
$F_y = 0 = -mg \cos \theta + N$

if $mg \sin \theta > \mu_s N$, must slide
Critical angle $\mu_s = \tan \theta_c$

Inclined plane with “Normal” and Frictional Forces

1. Static Equilibrium Case
2. Dynamic Equilibrium (Friction opposite velocity (down the incline))

Block weight is $mg$

Normal Force

$F_x = 0 = mg \sin \theta - f_k$
$F_y = 0 = -mg \cos \theta + N$

$f_k = \mu_k N = \mu_k mg \cos \theta$

$\mu_k = \tan \theta_c$ (only one angle)

Inclined plane with “Normal” and Frictional Forces

3. Dynamic case with non-zero acceleration
Result depends on direction of velocity

Weight of block is $mg$

Normal Force

$F_x = ma_x = mg \sin \theta \pm f_k$
$F_y = 0 = -mg \cos \theta + N$

at turnaround point, block is always motion although there is an infinitesimal point at which the velocity of the block passes through zero.

At this moment, depending on the static friction the block may become stuck.

Forces with rotation about a fixed axis

Key steps

1. Identify forces (i.e., a FBD)
2. Identify axis of rotation
3. Circular motion implies $a_r$ & $F_r = ma_r$
4. Apply conditions (position, velocity & acceleration)

Example

The pendulum

Consider a person on a swing:

When is the tension equal to the weight of the person + swing?

(A) At the top of the swing (turnaround point)
(B) Somewhere in the middle
(C) At the bottom of the swing
(D) Never, it is always greater than the weight
(E) Never, it is always less than the weight
Example

Gravity, Normal Forces etc.

at top of swing $v_t = 0$

\[ F_r = m \frac{v_t^2}{r} = T - mg \cos \theta \]

\[ T = mg \cos \theta \]

\[ F_r = ma = m \frac{v_t^2}{r} = T - mg \]

\[ T = mg + m \frac{v_t^2}{r} \]

\[ T < mg \]

Conical Pendulum (Not a simple pendulum)

\[ \sum F_r = ma_r = T \sin \theta \]

\[ \sum F_z = \cos \theta - mg \]

\[ T = mg \cos \theta \]

\[ a_r = g \tan \theta = \frac{v_t^2}{r} \]

\[ v_t = \left( \frac{gr \tan \theta}{2} \right)^{1/2} \]

\[ T = 2\pi \left( \frac{L \cos \theta}{g} \right)^{1/2} \]

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Another example of circular motion

Loop-the-loop 1

A match box car is going to do a loop-the-loop of radius $r$. What must be its minimum speed $v_t$ at the top so that it can manage the loop successfully?

\[ v_t = \left( \frac{gr \tan \theta}{2} \right)^{1/2} \]

\[ T = 2\pi \left( \frac{L \cos \theta}{g} \right)^{1/2} \]

\[ T = 2\pi \left( \frac{L \cos \theta}{g} \right)^{1/2} \]

Loop-the-loop 2

The match box car is going to do a loop-the-loop. If the speed at the bottom is $v_B$, what is the normal force, $N$, at that point?

\[ F_r = ma = mv_B^2/r = N - mg \]

\[ N = mv_B^2/r + mg \]

Hint: The car is constrained to the track.
Once again the car is going to execute a loop-the-loop. What must be its minimum speed at the bottom so that it can make the loop successfully?

This is a difficult problem to solve using just forces. We will skip it now and revisit it using energy considerations later on...

Orbiting satellites

\[ v_t = (gr)^{\frac{1}{2}} \]

Net Force:

\[ ma = mg = \frac{mv_t^2}{r} \]

\[ gr = v_t^2 \]

The only difference is that g is less because you are further from the earth's center!

Geostationary orbit

- The radius of the Earth is ~6000 km but at 36000 km you are ~42000 km from the center of the earth.

- \( F_{\text{gravity}} \) is proportional to \( 1/r^2 \) and so little g is now ~10 m/s\(^2\) / 50

\[ v_t = (0.20 \times 42000000)^{\frac{1}{2}} \text{ m/s} = 3000 \text{ m/s} \]

- At 3000 m/s, period \( T = \frac{2\pi r}{v_t} = \frac{2\pi \times 42000000}{3000} \text{ sec} = 90000 \text{ sec} = 90000 \text{ s}/3600 \text{ s/hr} = 24 \text{ hrs} \)

- Orbit affected by the moon and also the Earth's mass is inhomogeneous (not perfectly geostationary)

- Great for communication satellites

(1st pointed out by Arthur C. Clarke)

Recap

Assignment: HW5

For Tuesday: Read Chapter 7.1-7.4