Lecture 8

Today:

- Review session

Chapter 1: Concept of Motion

Chapter 2: 1D Kinematics

Chapter 3: Vector and Coordinate Systems

Chapter 4: Dynamics I, Two-dimensional motion

Exam will reflect most key points (but not all)

25-30% of the exam will be more conceptual

70-75% of the exam is problem solving

Chapter 1

Dimensions and Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Area (A)</th>
<th>Volume (V)</th>
<th>Length (L)</th>
<th>Time (T)</th>
<th>Acceleration (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>L^2</td>
<td>L^3</td>
<td>L</td>
<td>T</td>
<td>L/T^2</td>
</tr>
<tr>
<td>SI units</td>
<td>m^2</td>
<td>m^3</td>
<td>m/s</td>
<td>m/s^2</td>
<td></td>
</tr>
<tr>
<td>U.S. customary</td>
<td>ft^2</td>
<td>ft^3</td>
<td>ft/s</td>
<td>ft/s^2</td>
<td></td>
</tr>
</tbody>
</table>

Chapter 2

1D Kinematic: Motion in one dimension

- Position, displacement, velocity, acceleration
- Average velocity & acceleration
- Instantaneous velocity & acceleration
- Average and instantaneous speed
- Motion diagram
- Motion graphs (x vs. t, v vs. t and a vs. t)

Given the displacement-time graph (a)

- The velocity-time graph is found by measuring the slope of the position-time graph at every instant.
- The acceleration-time graph is found by measuring the slope of the velocity-time graph at every instant.

Chapter 3

Important Concepts

- Vectors & Scalar
- Vector addition and subtraction (graphical or components)
- Multiplication of a vector by a scalar
- Conversion between Cartesian & Polar coordinates
Decomposing vectors

Any vector can be resolved into components along the x and y axes.

\[ r = x + y \]

\[ \theta = \tan^{-1} \left( \frac{y}{x} \right) \]

\[ r_x = x \cos \theta \]
\[ r_y = x \sin \theta \]

\[ a_x = a \cos \theta \]
\[ a_y = a \sin \theta \]

\[ v_x = v \cos \theta \]
\[ v_y = v \sin \theta \]

\[ \theta = \tan^{-1} \left( \frac{\bar{v}_y}{\bar{v}_x} \right) \]

\[ r = \sqrt{x^2 + y^2} \]

\[ a = \sqrt{a_x^2 + a_y^2} \]

Chapter 3

Important Concepts

- Unit Vectors
- Component (x, y, z)
- Magnitude
- Direction

Using Vectors

Components

- Components are parallel to the x and y axes.
- Component along x is given by: \[ \bar{a}_x = a \cos \theta \]
- Component along y is given by: \[ \bar{a}_y = a \sin \theta \]

Chapter 4

General Principles

- Uniform circular motion
  - Angular velocity \( \omega = \frac{v}{r} \)
  - Angular acceleration \( \alpha = \frac{a}{r} \)

Important Concepts

- Uniform circular motion
  - Angular velocity \( \omega = \frac{v}{r} \)
  - Angular acceleration \( \alpha = \frac{a}{r} \)

Applications

- Basic skills tested
  - Count number of sig. figures
  - Apply addition/subtraction, multiplication/division rules for sig. figures.
  - Basic vector operations
  - Interpret x-t, v-t, a-t graphs
  - Use kinematical equation to convert among x, t, v, a.
  - For circular motion, relate radial acceleration to \( v, r, \omega \)
  - For uniform circular motion, calculate \( T \) from \( r \) and \( v \).
  - Decompose a vector quantities into component parallel & perpendicular components.
  - For motion on a curved path resolve acceleration into parallel & perpendicular components.
  - Calculate relative velocity when switching from one reference frame to another.
  - Free Fall and Projectile (1D and 2D)
  - Deduce flight time, maximum height, range, etc.
Welcome to Wisconsin

1. You are traveling on a two lane highway in a car going a speed of 20 m/s (~45 mph). You are notice that a deer that has jumped in front of a car traveling at 40 m/s and that car avoids hitting the deer but does so by moving into your lane! There is a head on collision and your car travels a full 2 m before coming to rest. Assuming that your acceleration in the crash is constant. What is your acceleration in terms of the number of g’s (assuming g is 10 m/s^2)?

   1. Draw a Picture
   1. Key facts (what is important, what is not important)
   1. Attack the problem

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Welcome to Wisconsin

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   1. Key facts: v\_initial = 20 m/s, after 2 m your v = 0.

   \[
   x = x\_initial + v\_initial \Delta t + \frac{1}{2} a \Delta t^2
   \]

   \[
   x = x\_initial - 2 m = v\_initial \Delta t + \frac{1}{2} a \Delta t^2
   \]

   \[
   v = v\_initial + a \Delta t
   \]

   \[
   v = -\frac{v\_initial}{a} \Rightarrow a = -\frac{v\_initial}{\Delta t}
   \]

   \[
   -2 m = v\_initial (-\frac{v\_initial}{a}) + \frac{1}{2} a (-\frac{v\_initial}{a})^2
   \]

   \[
   -2m = -\frac{1}{2}v\_initial^2 / a \Rightarrow a = (20 m/s)^2 / 2 m = 100 m/s^2
   \]

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Analyzing motion plots

1. The graph is a plot of velocity versus time for an object. Which of the following statements is correct?
   A. The acceleration of the object is zero.
   B. The acceleration of the object is constant.
   C. The acceleration of the object is positive and increasing in magnitude.
   D. The acceleration of the object is negative and decreasing in magnitude.
   E. The acceleration of the object is positive and decreasing in magnitude.

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Short word problems

1. After breakfast, I weighed myself and the scale read 588 N. On my way out, I decide to take my bathroom scale in the elevator with me. What does the scale read as the elevator accelerates downwards with an acceleration of 1.5 m/s^2?

   A bear starts out and walks 1\text{st} with a velocity of 0.60 \text{j} m/s for 10 seconds and then walks at 0.40 \text{i} m/s for 20 seconds. What was the bear’s average velocity on the walk? What was the bear’s average speed on the walk (with respect to the total distance travelled)?

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Conceptual Problem

The pictures below depict cannonballs of identical mass which are launched upwards and forward. The cannonballs are launched at various angles above the horizontal, and with various velocities, but all have the same vertical component of velocity.

Which of the following correctly ranks the time the balls are in the air, from shortest to longest?

a) 1, II, III
b) 3, II, I
c) 1, II, III
d) All the balls are in the air for the same amount of time

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Graphing problem

The figure shows a plot of velocity vs. time for an object moving along the x-axis. Which of the following statements is true?

A. The average acceleration over the 11.0 second interval is -0.36 m/s^2
B. The instantaneous acceleration at t = 5.0 s is -4.0 m/s^2
C. Both A and B are correct.
D. Neither A nor B are correct.

Note: \( \Delta x \neq \frac{1}{2} a_{avg} \Delta t^2 \)
Sample Problem

A physics student on Planet Exidor throws a ball that follows the parabolic trajectory shown. The ball’s position is shown at one-second intervals until \( t = 3 \) s. At \( t = 1 \) s, the ball’s velocity is \( \mathbf{v} = (2 \mathbf{i} + 2 \mathbf{j}) \) m/s.

a. Determine the ball’s velocity at \( t = 0 \) s, 2 s, and 3 s.

b. What is the value of \( g \) on Planet Exidor?

---

Sample Problem

A friend decides to try out a new slingshot. Standing on the ground he finds out the best he can do is to shoot a stone at an angle of 45° above the horizontal at a speed of 20.0 m/s. The stone flies forward and, at the top of the trajectory, the stone just hits a vertical wall a small distance, 1.25 m, below the top edge. (Neglect air resistance and assume that the acceleration due to gravity is exactly 10 m/s² downward. Also note \( \cos 45° = \sin 45° = 0.7071 \))

A. What is the stone’s velocity just before it hits the wall?

\[
\mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j}
\]

\[
v_x = v \cos \theta = 20 \times 0.7071 = 14.1 \text{ m/s}
\]

B. How high above the level of the slingshot does the stone rise?

\[
h = (v_y)^2 / (2g)
\]

\[
h = (20 \times 0.7071)^2 / (2 \times 10) = 5.41 \text{ m}
\]

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Ferris Wheel Physics

A Ferris wheel, with radius 10.0 m, undergoes 5 full clockwise revolutions in 3 minutes.

- What is the period? \( T = 180 \text{ sec} / 5 = 36 \text{ sec} \)
- What is the angular velocity? \( \omega = 2\pi / 36 \text{ sec} = 0.174 \text{ rad/s} \)
- After these five revolutions you are at the very bottom of the wheel.

What is the radial acceleration (direction and magnitude)?

\[
a_r = \omega^2 r = (0.174)^2 \times 10 = 0.303 \text{ m/s}^2
\]

Just then your speed starts to increase. An accelerometer reads a value of 0.50 m/s².

What is the radial acceleration (direction and magnitude)?

\[
a^2 = a_r^2 + a_i^2
\]

\[
\mathbf{a} = (-0.46 \mathbf{i} + 0.30 \mathbf{j}) \text{ m/s}^2
\]

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Conceptual Problem

A person initially at point \( P \) in the illustration stays there a moment and then moves along the axis to \( Q \) and stays there a moment. She then runs quickly to \( R \), stays there a moment, and then strolls slowly back to \( P \). Which of the position vs. time graphs below correctly represents this motion?
Another question to ponder

How high will it go?

One day you are sitting somewhat pensively in an airplane seat and notice, looking out the window, one of the jet engines running at full throttle. From the pitch of the engine you estimate that the turbine is rotating at 3000 rpm and, give or take, the turbine blade has a radius of 1.00 m. If the tip of the blade were to suddenly break off (it occasionally does happen with negative consequences) and fly directly upwards, then how high would it go (assuming no air resistance and ignoring the fact that it would have to penetrate the metal cowling of the engine.)

\[
\begin{align*}
\omega &= 3000 \text{ rpm} = (3000 \times \frac{2\pi}{60}) \text{ rad/s} = 314 \text{ rad/s} \\
\tau &= 1.00 \text{ m} \\
v_0 &= \omega r = 314 \text{ m/s (~650 mph!)} \\
h &= h_0 + v_0 t - \frac{1}{2} g t^2 \\
h &= 0 = v_0 - g t \\
t &= \frac{v_0}{g} \\
So \quad h &= \frac{v_0}{g} \left( -\frac{1}{2} g t^2 = \frac{1}{2} v_0^2 / g = 0.5 \times 314^2 / 9.8 = 5 \text{ km} \right) \\
or &\sim 3 \text{ miles}
\end{align*}
\]

Sample exam problem

Two push carts start out from the same x position (at \( t = 0 \) seconds) on a track and have x velocity plots as shown below.

(a) For cart A, what is the average speed in the first five seconds?
(b) Do these carts ever again have a common x position (or positions) and, if so, when does this occur?

A day at the airport

You are standing on a moving walkway at an airport. The walkway is moving at 1.0 m/s. Just now you notice a friend standing 20 m ahead of you and you wish to catch up to your friend before they get to the end of the moving walkway. They are 10 m away from the end.

A. If you move towards your friend with constant acceleration, what must that acceleration be if you are to just meet up with your friend as they reach the end of the moving walkway?

The time is set by your friend and their distance to the end.

The distance you must cover is 20 m

\[ t = \frac{10 \text{ m}}{1 \text{ m/s}} = 10 \text{ sec} \]

The distance you must cover is 20 m

\[ 20 \text{ m} = \frac{1}{2} a t^2 = \frac{1}{2} a 100 \text{ s}^2 \]

\[ a = 0.40 \text{ m/s}^2 \]

B. \( v = 1.0 \text{ m/s} + at = 5.0 \text{ m/s} \)