Lecture 4

Today: Chapter 3

- Introduce scalars and vectors
- Perform basic vector algebra (addition and subtraction)
- Interconvert between Cartesian & Polar coordinates

**A race**

According to our dynamical equations

- \( v(t) = v_i + a t \)
- \( v(t) = v_i + a t \)

A trivial solution is \( \theta = 0^\circ \) and the race requires 2 seconds.

Horizontal travel

- Constant velocity so \( t = \frac{d}{v} = \frac{2 \times 9.8 \text{ m}}{9.8 \text{ m/s}} \)
- \( t = 2 \cos \theta \) seconds

Incline travel

- \( 9.8 \text{ m} = 9.8 \text{ m/s} (t_2/2) + \frac{1}{2} g \sin \theta \left( \frac{t_2}{2} \right)^2 \)
- \( t_2 = \frac{-4 \pm \sqrt{16 + 32 \sin \theta}}{2 \sin \theta} \)

Solve graphically

If \( \theta < 36^\circ \) then the incline is faster
If \( \theta > 36^\circ \) then the horizontal track is faster

Solving graphically is easier

\[ t_1 = 2 \cos \theta = \frac{-2 + \sqrt{4 + 8 \sin \theta}}{\sin \theta} \]

A science project

- You drop a bus off the Willis Tower (442 m above the side walk). It so happens that Superman flies by at the same instant you release the bus. Superman is flying down at 35 m/s.
- How fast is the bus going when it catches up to Superman?

Solving analytically is a challenge

\[ 2 \cos \theta = \frac{-2 + \sqrt{4 + 8 \sin \theta}}{\sin \theta} \]

\[ 2 \cos \theta + \sin \theta \cos^2 \theta - 2 = 0 \]

Let \( z = \cos \theta \) and \( \sqrt{1 - z^2} = \sin \theta \)

Solve: \[ z^6 - z^4 + 4z^2 - 8z + 4 = 0 \]

A sixth order polynomial, (if \( z = 1 \), \( 1 - 1 + 4 - 8 + 4 = 0 \))
A “science” project

1. You drop a bus off the Willis Tower (442 m above the side walk). It so happens that Superman flies by at the same instant you release the car. Superman is flying down at 35 m/s.

1. How fast is the bus going when it catches up to Superman?

1. Draw a picture

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Home Exercise: Welcome to Wisconsin

1. You are traveling on a two lane highway in a car going a speed of 20 m/s (45 mph). You are notice that a deer that has jumped in front of a car in the opposite lane traveling at 40 m/s (90 mph) and that car avoids hitting the deer but does so by moving into your lane! There is a head on collision and your car travels a full 2m before coming to rest. Assuming that your acceleration in the crash is constant. What is your acceleration in terms of the number of g’s (assuming g is 10 m/s²)?

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Welcome to Wisconsin

1. You are traveling on a two lane highway in a car going a speed of 20 m/s. You are notice that a deer that has jumped in front of a car traveling at 40 m/s and that car avoids hitting the deer but does so by moving into your lane! There is a head on collision and your car travels a full 2m before coming to rest. Assuming that your acceleration in the crash is constant. What is your acceleration in terms of the number of g’s (assuming g is 10 m/s²)?

1. Draw a Picture

1. Key facts: what is important, what is not important

1. Attack the problem

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Coordinate Systems and Vectors

1. In 1 dimension, only 1 kind of system,

   - Linear Coordinates (x) +/-

1. In 2 dimensions there are two commonly used systems,

   - Cartesian Coordinates (x,y)
   - Circular Coordinates (r,θ)

1. In 3 dimensions there are three commonly used systems,

   - Cartesian Coordinates (x,y,z)
   - Cylindrical Coordinates (r,θ,z)
   - Spherical Coordinates (r,θ,φ)
Scalars and Vectors

- A scalar is an ordinary number.
- Has magnitude (+ or -), but no direction
- May have units (e.g. kg) but can be just a number
- Represented by an ordinary character

Examples:
- mass (m, M) kilograms
- distance (d, s) meters
- spring constant (k) Newtons/meter

Vectors act like...

- Vectors have both magnitude and a direction
- Vectors: position, displacement, velocity, acceleration
- Magnitude of a vector \( \mathbf{A} = |\mathbf{A}| \)

1. For vector addition or subtraction we can shift vector position at will (NO ROTATION)
2. Two vectors are equal if their directions, magnitudes & units match.

\[ \mathbf{A} = \mathbf{C}, \quad \mathbf{B} \neq \mathbf{C} \]

Vectors look like...

- There are two common ways of indicating that something is a vector quantity:
  - Boldface notation: \( \mathbf{A} \)
  - "Arrow" notation: \( \mathbf{A} \)

Scalars and Vectors

- A scalar can't be added to a vector, even if they have the same units.
- The product of a vector and a scalar is another vector in the same "direction" but with modified magnitude

\[ \mathbf{A} = -0.75 \mathbf{B} \]

Exercise

Vectors and Scalars

While I conduct my daily run, several quantities describe my condition

Which of the following is cannot be a vector?

A. my velocity (3 m/s)
B. my acceleration downhill (30 m/s²)
C. my destination (the lab - 100,000 m east)
D. my mass (150 kg)

Vectors and 2D vector addition

1. The sum of two vectors is another vector.

\[ \mathbf{A} = \mathbf{B} + \mathbf{C} \]
2D Vector subtraction

Vector subtraction can be defined in terms of addition.

\[ \mathbf{B} - \mathbf{C} = \mathbf{B} + (-1)\mathbf{C} \]

Different direction and magnitude!

Unit Vectors

A Unit Vector points: a length 1 and no units
Gives a direction.
Unit vector \( \mathbf{u} \) points in the direction of \( \mathbf{U} \)
Often denoted with a “hat”: \( \mathbf{u} = \hat{\mathbf{u}} \)

<table>
<thead>
<tr>
<th>( \hat{i} )</th>
<th>( \hat{j} )</th>
<th>( \hat{k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point in the direction of the ( x ), ( y ) and ( z ) axes.</td>
<td></td>
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\[ \mathbf{U} = |\mathbf{U}| \hat{\mathbf{u}} \]

Useful examples are the cartesian unit vectors \( \{ \hat{i}, \hat{j}, \hat{k} \} \) or

\[ \mathbf{R} = r_\mathbf{x} \hat{i} + r_\mathbf{y} \hat{j} + r_\mathbf{z} \hat{k} \]

or

\[ \mathbf{R} = x \hat{i} + y \hat{j} + z \hat{k} \]

Unit Vector points: a length 1 and no units
Gives a direction.
Unit vector \( \mathbf{u} \) points in the direction of \( \mathbf{U} \)

Example Vector Addition

Vector \( \mathbf{A} = \{0,2,1\} \)
Vector \( \mathbf{B} = \{3,0,2\} \)
Vector \( \mathbf{C} = \{1,-4,2\} \)

What is the resultant vector, \( \mathbf{D} \), from adding \( \mathbf{A} + \mathbf{B} + \mathbf{C} \)?

A. \( \{3,-4,2\} \)
B. \( \{4,-2,5\} \)
C. \( \{5,-2,4\} \)
D. None of the above

Converting Coordinate Systems (Decomposing vectors)

In polar coordinates the vector \( \mathbf{R} = (r, \theta) \)
In Cartesian the vector \( \mathbf{R} = (r_\mathbf{x}, r_\mathbf{y}) = (x, y) \)
We can convert between the two as follows:

\[ r = \sqrt{x^2 + y^2} \]
\[ \theta = \tan^{-1}(y/x) \]

\( x = r \cos \theta \)
\( y = r \sin \theta \)
\( x = r_\mathbf{x} \)
\( y = r_\mathbf{y} \)

Motion in 2 or 3 dimensions

Position \( \mathbf{r}, t_f \) and \( \mathbf{r}, t_f \)
Displacement \( \Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i \)
Velocity (avg.) \( \mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{\Delta t} \)
Acceleration (avg.) \( \mathbf{a}_{\text{avg}} = \frac{\Delta \mathbf{v}}{\Delta t} \)
Physics 201 – Lecture 4

Kinematics

In 2-dim. position, velocity, and acceleration of a particle:

\[ r = x \mathbf{i} + y \mathbf{j} \]
\[ v = v_x \mathbf{i} + v_y \mathbf{j} \quad (i, j \text{ unit vectors}) \]
\[ a = a_x \mathbf{i} + a_y \mathbf{j} \]

\[ x = x(\Delta t) \quad y = y(\Delta t) \]
\[ v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \]
\[ a_x = \frac{d^2x}{dt^2} \quad a_y = \frac{d^2y}{dt^2} \]

With, if constant x accel.: \( x(\Delta t) = x_0 + v \Delta t + \frac{1}{2} a \Delta t^2 \)
With, if constant y accel.: \( y(\Delta t) = y_0 + v \Delta t + \frac{1}{2} a \Delta t^2 \)

All this complexity is hidden away in

\[ r = \mathbf{r}(\Delta t) \quad v = \frac{d\mathbf{r}}{dt} \quad a = \frac{d^2\mathbf{r}}{dt^2} \]

Kinematics

The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:

\[ r = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \]
\[ v = v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k} \quad (i, j, k \text{ unit vectors}) \]
\[ a = a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k} \]

\[ x = x(\Delta t) \quad y = y(\Delta t) \quad z = z(\Delta t) \]
\[ v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt} \]
\[ a_x = \frac{d^2x}{dt^2} \quad a_y = \frac{d^2y}{dt^2} \quad a_z = \frac{d^2z}{dt^2} \]

With, if constant accel., e.g. \( x(\Delta t) = x_0 + v \Delta t + \frac{1}{2} a \Delta t^2 \)

All this complexity is hidden away in

\[ r = \mathbf{r}(\Delta t) \quad v = \frac{d\mathbf{r}}{dt} \quad a = \frac{d^2\mathbf{r}}{dt^2} \]

Lecture 4

Assignment: Read Chapter 4.1 to 4.3