1) At the Earth we have 1.36 kW/m². We can find the total power radiated by the sun by imagining a sphere of radius equal to the Earth-Sun separation: \( P = (1.36 \text{ kW/m}^2) \cdot 4\pi R_e^2 \). Then the power per unit area at the Sun's surface is \( P/4\pi R_s^2 \Rightarrow \)

\[
\text{power per unit area} = (1.36 \text{ kW/m}^2) \frac{(1.5 \times 10^8 \text{ m})^2}{(6.96 \times 10^8 \text{ m})^2} = 6.32 \times 10^7 \text{ W/m}^2 = \sigma T^4
\]

\[
T = \left( \frac{6.32 \times 10^7 \text{ W/m}^2}{\sigma} \right)^{\frac{1}{4}} = \left( \frac{6.32 \times 10^7 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4} \right)^{\frac{1}{4}}
\]

\[
T = 5777 \text{ K}
\]

From the Wien displacement law (p.9)

\[
\lambda_{\text{max}} = 2899 \times 10^{-6} \text{ m.K}/T = 502 \text{ nm}
\]

(b) The total power (from above) is

\[
P = (1.36 \times 10^3 \text{ W/m}^2) \cdot 4\pi (1.5 \times 10^8 \text{ m})^2 = 3.85 \times 10^{26} \text{ W}
\]

2) For circular orbits \( |\omega| = \frac{v^2}{r} \) so we have

\[
m \frac{v^2}{r} = kr
\]

According to Bohr, the angular momentum is quantized:

\[
L = mrv = nh \Rightarrow v = \frac{nh}{mr}
\]

Substitute into (1)

\[
m \left( \frac{nh}{mr} \right)^2 = kr^2
\]

\[
\frac{n^2h^2}{kmr^4} = kr
\]

\[
r^2 = \frac{nh}{\sqrt{km}}
\]

The total energy is
\[ E = \frac{1}{2}mv^2 + \frac{1}{2}kr^2 \]

From (i) \( mv^2 = kr^2 \) so \( E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2 = kr^2 \)

\[ E = k \frac{n^2}{\sqrt{2m}} = \frac{n^2}{\sqrt{2m}} \]

3) (a) We have energies \( E = -\frac{mc^2}{\alpha^2} \frac{1}{n^2} = -13.6\text{eV}/n^2 \). So the first \( \Delta E \) is \( 13.6\text{eV}/1 - 13.6\text{eV}/4 = 10.2\text{eV} \) giving \( \frac{\Delta E}{E_1} = 0.75 \)

(b) i) I'm assuming the pendulum is released from rest at 60°. Then it's energy is \( E = mgh = mgl(1-\cos 60°) \)

\[ = (0.005\text{kg})(9.8\text{m/s}^2)(2\text{m})(0.5) \]

\[ = 4.9 \times 10^{-2} \text{J} \]

ii) For small oscillations the frequency is \( f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} = 0.352 \text{rad/s}. \) The motion of the pendulum is sinusoidal - just like a mass on a spring - and the quantization rule is the same. \( E = nhf \)

so \( \Delta E = hf = (6.626 \times 10^{-34} \text{J}\cdot\text{s})(0.352 \text{rad/s}) = 2.33 \times 10^{-34} \)

giving \( \frac{\Delta E}{E_1} = 4.76 \times 10^{-33} \)

(c) In (b) the spacing between states is very small compared to \( E \), so the energy quantization is not important. In the hydrogen atom it is very important.
4) According to the Bohr model, the orbits have radius \( r = \frac{4\pi^2 e^2}{\varepsilon_0^2 m n^2} \) and velocity \( v = \left( \frac{e^2}{4\pi^2 \varepsilon_0 \hbar} \right)^{\frac{1}{2}} n \) so the orbit frequency is
\[
f = \frac{v}{\text{orbit circumference}} = \frac{2\pi r}{\text{orbit circumference}} = \left( \frac{\hbar}{2\pi} \right) \left( \frac{e^2}{4\pi^2 \varepsilon_0} \right)^{\frac{1}{2}} \frac{m}{\hbar^3} \frac{1}{n^3}
\]
For a quantum jump from state \( n+1 \) to state \( n \) the radiation frequency would be
\[
\nu = E_{n+1} - E_n
\]
\Rightarrow
\[
\nu = \frac{1}{\hbar} \left[ \left( \frac{e^2}{4\pi^2 \varepsilon_0} \frac{1}{2} \frac{m c^2}{n^2} \right) m c^2 \right] \left[ \frac{1}{n^2} - \frac{1}{(n+1)^2} \right]
\]
For large \( n \)
\[
\frac{1}{(n+1)^2} = \frac{1}{n^2 (\frac{1}{n} + 1)^2} = \frac{1}{n^2}\left(1 + \frac{1}{n}\right)^{-2}
\]
\[
\approx \frac{1}{n^2} \left(1 - \frac{2}{n} + \ldots\right)
\]
So
\[
\nu = \frac{1}{2\pi \hbar} \left( \frac{e^2}{4\pi^2 \varepsilon_0} \right)^{\frac{1}{2}} \frac{m}{\hbar^2} \left[ \frac{1}{n^2} - \left( \frac{1}{n} \right)^2 \left(1 - \frac{2}{n} + \ldots\right) \right]
\]
\[
\nu \approx \frac{1}{2\pi \hbar} \left( \frac{e^2}{4\pi^2 \varepsilon_0} \right)^{\frac{1}{2}} \frac{m}{\hbar^3} \frac{1}{n^3}
\] is same as \( f \).

5) We need to do \( \lambda = \frac{\hbar}{\nu} \). All the examples are non-relativistic so when the energy is given \( E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE} \). I will use
\[
\lambda = \frac{hc}{pc} \quad \text{so I want} \quad pc = \sqrt{2E mc^2}
\]
(a) \[
pc = \sqrt{2(54eV)(5.11 \times 10^5 eV)} = 7430 eV \quad \lambda = \frac{1240 eV \text{ nm}}{7430 eV} = 0.167 \text{ nm}
\]
(b) \[
pc = \sqrt{2(70 MeV)(938 MeV)} = 362.4 \text{ MeV} \quad \lambda = 3.42 \times 10^{-6} \text{ nm} = 3.42 \text{ fm}
\]
(c) \[
p = m v = (0.1 \text{ kg})(1200 \text{ m/s}) = 120 \text{ kg \cdot m/s}
\]
\[
\lambda = \frac{\hbar}{p} = \frac{6.62 \times 10^{-34} \text{ J \cdot s}}{120 \text{ kg \cdot m/s}} = 5.5 \times 10^{-36} \text{ m}
\]
6) Postponed.

7) We have \( E^2 = (pc)^2 + (mc^2)^2 \). Substitute \( E = \hbar \omega \) and \( p = \hbar k \) and then solve for \( \omega \)

\[
(h \omega)^2 = [(hkc)^2 + (mc^2)^2]
\]

\[
\omega = \frac{1}{\hbar} \left[ (hkc)^2 + (mc^2)^2 \right]^{\frac{1}{2}}
\]

\[
\frac{dw}{dk} = \frac{1}{\hbar} \left( \frac{1}{2} \right) \left[ (hkc)^2 + (mc^2)^2 \right]^{-\frac{1}{2}} \sqrt{h^2 c^2 k}
\]

\[
= \frac{hkc^2}{\left[ (hkc)^2 + (mc^2)^2 \right]^{\frac{1}{2}}} = \frac{pc^2}{\left[ (pc)^2 + (mc^2)^2 \right]^{\frac{1}{2}}} = \frac{pc^2}{E}
\]

\[
\Rightarrow \quad \frac{dw}{dk} = \frac{8 \, mv \, c^2}{8 \, mc^2} = v
\]