14) The probability current for the wave function $\psi(x) = Ae^{ikx} + Be^{-ikx}$ is $J = \frac{\hbar}{im}(|A|^2 - |B|^2)$.
    
    (a) Find the corresponding probability current for $\psi(x) = Ce^{\alpha x} + De^{-\alpha x}$ where $\alpha$ is real but $C$ and $D$ are complex.
    
    (b) Suppose $\psi(x) = Ae^{ikx} + Be^{-ikx}$ for $x < 0$ and $\psi(x) = Ce^{\alpha x} + De^{-\alpha x}$ for $x > 0$. Match the two wave functions to get $C$ and $D$ in terms of $A$ and $B$, and then show that the probability currents for $x > 0$ and $x < 0$ are the same.

15) Zettili Exercise 4.3.

16) A particle has a wave function

$$\psi(x) = \begin{cases} A \cos \frac{\pi}{2a}x & \text{for } |x| < a \\ 0 & \text{for } |x| > a. \end{cases}$$

Find the uncertainty product $\Delta p \Delta x$.

17) The operators $\hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x}$ and $\hat{x} = x$ (i.e. multiply by $x$) do not commute. Show that $\hat{p}\hat{x} - \hat{x}\hat{p}$ acting on any function $f(x)$ is identical to $\frac{\hbar}{i} f(x)$. (Since this is true for all $f(x)$ we write $\hat{p}\hat{x} - \hat{x}\hat{p} = \frac{\hbar}{i}$.)

18) An operator $Q$ is linear if and only if $Q(\psi_1 + \psi_2) = Q\psi_1 + Q\psi_2$ and $Q(a\psi) = aQ\psi$. Determine which of the following operators are linear:

(a) $Q_1\psi = x \frac{\partial}{\partial x} \psi$.
(b) $Q_2\psi = \lambda \psi^*$.
(c) $Q_3\psi = e^\psi$.
(d) $Q_4\psi = \int_{-\infty}^x x' \psi(x') \, dx'$.

19) A particle of mass $m$ is incident on a potential barrier like that shown in Figure 4.3 of the text. Find the transmission probability for the case in which $E$ is exactly $V_0$. To find $T$ I want you to write down $\psi$ in each region, match the solutions at $x = 0$ and $x = a$, and then solve for $F$ in terms of $A$ (using the notation from class).