6) Zettiti Exercise 1-38 – postponed from Homework 1.

8) The standard deviation, $\sigma_x$, of a probability distribution is defined as $\sigma_x^2 = \langle (x - \langle x \rangle)^2 \rangle$. Show that this is equivalent to $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$.

9) In class we obtained a Gaussian wave packet of the form

$$\psi(x,t) = A \left[ \frac{2}{a^2 + i\eta^2 t} \right] e^{i(k_0x - \omega_0 t)} e^{-(x-\beta t)^2/(a^2+i\eta^2 t)}.$$ 

(a) Find the probability distribution $P(x) = |\psi(x,t)|^2$.

(b) Show that the integral of $P(x)$ over all $x$ does not depend on time.

10) Suppose we add a constant (for example $E_0 = mc^2$) to our definition of the energy of a particle. What does that do to the frequency of the deBroglie wave? What happens to the phase velocity? What happens to the group velocity?

11) The energy of a particle of mass $m$ subject to a harmonic oscillator potential is

$$E = T + V = \frac{p^2}{2m} + \frac{1}{2} kx^2.$$ 

Since $T \propto p^2$ and $V \propto x^2$, $E$ can never be negative. But with the uncertainty principle one can show that small positive values of $E$ are also ruled out. Find the minimum value of $E$ consistent with the uncertainty principle $\Delta p \cdot \Delta x \geq \hbar/2$. [Hints: Write $\langle E \rangle$ in terms of $\langle p^2 \rangle$ and $\langle x^2 \rangle$ and then use the result of Problem 8 to express $\langle E \rangle$ in terms of the uncertainties.]

12) A beam of electrons is to be fired over a distance of $10^4$ km. If the size of the initial wave packet is 1 mm, what will be the size upon arrival if its kinetic energy is 10 eV?

13) We have said that the expectation value of the momentum can be calculated from

$$\langle p \rangle = \hbar \int_{-\infty}^{\infty} \phi^*(k) k \phi(k) \, dk.$$ 

In this problem we will find a way to obtain $\langle p \rangle$ directly from $\psi$. Step 1: Use Eq. (1.96) to express $\phi(k)$ (inside the integral) in terms of $\psi(x)$. Step 2: Replace

$$ke^{-ikx} \text{ by } -\frac{1}{i} \frac{\partial}{\partial x} e^{-ikx}.$$ 

Step 3: Integrate by parts (in the variable $x$). Assume that $\psi(x)$ goes to zero at $\pm \infty$, so that the integrated piece can be discarded. This brings a sign change and moves the derivative onto $\psi$. Step 4: Recognize that the remaining $k$ integral is exactly $\psi^*(x)$ – see Eq. (1.95). You should now have $\langle p \rangle$ written as an integral involving only $\psi$, $\psi^*$ and the “operator” $\hbar \frac{\partial}{\partial x}$. 