Physics 241

Fully Worked Out Examples for Exam 2


(a) You are asked by a journalist to estimate what change of sunlight on the Earth will cause an Ice Age. The journalist knows no engineering. You know the average temperature of the Earth in 2010 was 75F (25C = 298K) and that a change (lowering) of 7C would, if nothing else changed, cause an Ice Age. What percentage reduction of sunlight would cause an Ice Age?

Answer: Take the Sun as a blackbody with a surface temperature ($T_S$). The total energy from the Sun is $\frac{\sigma}{6} T_S^4$ (area of Sun). To answer the question, we must assume that the Earth temperature is entirely due to the absorbed sunlight. Lowering the Earth surface temperature by 7C is a reduction of 2.35%. So the new Sun surface temperature ($T_{\text{new}} = T_N$) would have:

$$\frac{\sigma T_N^4}{\sigma T_S^4} = \left(\frac{T_N}{T_S}\right)^4 = 0.975$$

$$\Rightarrow T_N = (0.994) T_S = 99.4\% \text{ of } T_S$$

So a reduction of 0.6% is enough.
(b) If an object blackbody object has an initial temperature \( T_I = 400 \text{K} \), what temperature will it have if the wavelength of maximum decreases by 15%?

Answer: Wien’s law says
\[ \lambda_{\text{max}} \propto \frac{1}{T} \]

\[ \Rightarrow \lambda_{\text{max}} T_I = \lambda_{\text{max}} T_F \]

\[ \Rightarrow T_F = \left( \frac{\lambda_{\text{max}}}{\lambda_{\text{max}}} \right) T_I = \left( \frac{1}{0.85} \right) (400) \]

\[ = 471 \text{K}. \]

2. A material is studied. Lecture 12 notes.

The photoelectric properties of a material are studied. Light of wavelength \( \lambda = 200 \text{nm} \) produces maximum outgoing electron.

Energy of 1.2eV.

(a) What is the work function?

\[ h\nu = 6.0 \text{eV} = (k\xi) + \text{(work function)} \]

\[ = (1.2) + \Phi \]

\[ \Rightarrow \Phi = 4.8 \text{eV} \]

(b) What light wavelength is photoelectric threshold?

\[ \lambda \propto \frac{1}{\nu} \Rightarrow \lambda_{\text{threshold}} = \left( \frac{6.0}{4.8} \right) (200) \]

\[ = 250 \text{nm}. \]
(c) The material is in a spherical vessel. The vessel is at electrical ground. Light of wavelength \( \lambda = 150 \) nm shines on the material. What voltage \( V_0 \) with respect to the vessel should be applied to the material so that no photoelectrons reach the vessel?

Answer:

Maximum photoelectron kinetic energy just outside the material surface:

\[
\hbar v - \Phi = \left[16.0 \times \frac{200}{150}\right] - 4.8 \text{ eV}
\]

\[
= 3.2 \text{ eV}
\]

So the voltage is 3.2 volts.

3. X-rays. Lecture 13 notes.

Suppose that x-rays of initial wavelength \( \lambda_I \) are Compton scattered at angle \( \theta \) and then Bragg scattered in 2nd order scattering at the same angle \( \theta \) from a crystal with a separation between scattering planes \( d = 0.025 \text{ m} \). What is \( \lambda_F \) and the initial photon energy? The angle \( \theta = 2^\circ \).
3. (cont.)

Compton scattering: \( \lambda - \lambda_I = \left( \frac{h}{mc} \right) (1 - \cos \theta) \)

Bragg scattering (2nd order): \( 2\lambda = 2d \sin \theta \)

\[ \Rightarrow \lambda = d \sin \theta \]

\[ \Rightarrow \lambda_I = \left[ d \sin \theta \right] - \left( \frac{h}{mc} \right) (1 - \cos \theta) \]

\[ \frac{h}{mc} = 2.4 \times 10^{-12} \text{ m} \]
\[ d = 2.5 \times 10^{-10} \text{ m} \]
\[ \theta = 2^\circ = \frac{\pi}{90} \text{ radian} \]

\[ \sin \theta \approx \theta \]
\[ 1 - \cos \theta \approx \frac{\theta^2}{2} \] so 2nd term is negligible

\[ \Rightarrow \lambda_I \approx d \left( \frac{\pi}{90} \right) = 8.7 \times 10^{-13} \text{ m} \]

\[ h\nu = \text{photon energy} = \frac{hc}{\lambda} = \left( \frac{6.63 \times 10^{-34}}{8.7 \times 10^{-13}} \right) \times 3 \times 10^8 \]

\[ = 2.3 \times 10^{-13} \text{ joule} \]

\[ \approx 1.4 \times 10^6 \text{ eV (X-ray)} \]
4. Suppose there is a negatively charged particle orbiting a proton. The rest mass of the negatively charged particle is 2.04 MeV/keV.
   (a) Determine the energy of the ground state.
   (b) Determine the energy of the 1st Balmer line.

Answer:
(a) \( E_{\text{binding}} \propto \text{mass} \rightarrow E_0 = (-13.6 \text{ eV})(4.00) \)
\[ = -54.4 \text{ eV} \]

(b) 1st Balmer line:
Energy (hydrogen) = +13.6 eV \( \left( \frac{1}{3} - \frac{1}{4} \right) \)
\[ = (413.6 \text{ eV}) \left( \frac{1}{2} - \frac{1}{3} \right) \]
\[ = \text{for this "atom"} \]
Energy = \( (54.4) \left( \frac{1}{2} - \frac{1}{3} \right) = 7.62 \text{ eV} \)

5. You build an \( \alpha \)-particle source that provides \( 1.00 \times 10^4 \) \( \alpha \)-particles per second on target. You want to measure the fraction scattered between 30° and 45° to an accuracy of 0.5%. How long must the experiment run? The gold foil is 2.0μm thick.

Answer: Refer to Lecture 15 notes.

P.8: \( I_0 = 1.00 \times 10^4 \)

P.9: \( \frac{dN}{dt} = \left[1.00 \times 10^4\right] \left[1.18 \times 10^{-5}\right] \Delta \sigma \)
\[ \Delta \sigma = (4\pi \times 1.29) \text{ barns} \left( \frac{1}{\sin^2(\theta_1/2)} - \frac{1}{\sin^2(\theta_2/2)} \right) \]

\[ \frac{\theta_1}{2} = 15^\circ = 0.262 \text{ radian} \]

\[ \frac{\theta_2}{2} = 22.5^\circ = 0.393 \text{ radian} \]

\[ \Rightarrow \left[ \text{ } \right] = \left[ 14.9 - 4.83 \right] = 8.07 \]

\[ \Rightarrow \Delta \sigma = 131 \text{ barn} \]

\[ \Rightarrow \frac{dN}{dt} = 1.55 \times 10^4 = 15.5 \times N \text{ particles scattered per second} \]

Desired: Accuracy of 0.5\% \Rightarrow \frac{1}{\sqrt{N}} = \frac{5 \times 10^{-3}}{N}

\[ \Rightarrow \sqrt{N} = 2 \times 10^2 \Rightarrow N = 4 \times 10^4 \]

\[ \Rightarrow \text{time} = \frac{4 \times 10^4}{1.55 \times 10^4} = 2.58 \times 10^{-3} \text{ seconds} \]

6. Refer to Lecture 17 Notes

Suppose I want to use a \( \mu \)-meson in an experiment. I want a \( \mu \)-meson source with a wavelength of 1.20 nm. What kinetic energy must the \( \mu \)-mesons have?

Answer:

\[ P = \frac{\hbar}{\lambda} \text{ or } \lambda = \frac{\hbar}{P} \Rightarrow P = \frac{6.63 \times 10^{-34}}{1.20 \times 10^{-9}} = 5.53 \times 10^{-25} \text{ joule} \]

\[ E = \frac{P^2}{2m} = \left( \frac{5.53 \times 10^{-25}}{2 \times 1.88 \times 10^{-28}} \right)^2 \text{ joule} = 8.13 \times 10^{-22} \text{ joule} \]

or \[ E = \frac{8.13 \times 10^{-22}}{1.6 \times 10^{-19}} \text{ eV} = 5.08 \times 10^{-3} \text{ eV} \]
7. Refer to Lecture 18 notes.

(a) You want to measure the lifetime of an atomic excited state. The excited state emits light of wavelength \( \lambda = 500 \text{ nm} \) when it decays to the ground state. Your measuring instrument has a resolving power of 50,000. What is the smallest lifetime you can measure before it is below what your instrument can detect?

\[ \frac{\Delta \lambda}{\lambda} = 50,000 = \frac{\nu}{\Delta \nu} = \frac{E}{\Delta E} \]

\[ E = h \nu = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) (3 \times 10^8)}{5 \times 10^{-7}} = 3.98 \times 10^{-19} \text{ J} \]

\[ \Delta E = \frac{3.98 \times 10^{-19}}{5 \times 10^4} = 7.96 \times 10^{-24} \text{ J} \]

\[ \Delta t \geq \frac{\Delta E}{\Delta E} > \frac{1}{2} = 0.528 \times 10^{-34} \text{ s} \]

\[ \Delta t \geq \frac{5.28 \times 10^{-35}}{7.96 \times 10^{-24}} = 6.63 \times 10^{-12} \text{ s} \]

is the smallest (shortest) lifetime you can measure with your equipment.

(b) Suppose \( \lambda = 250 \text{ nm} \). All else the same as (a). What is the smallest measurable lifetime?

\[ E = 7.96 \times 10^{-19} \text{ J} \]

\[ \Rightarrow \Delta E = 1.59 \times 10^{-23} \text{ J} \]

\[ \Rightarrow \Delta t \geq 3.32 \times 10^{-12} \text{ s} \]