Vertical Spring - Gravity just SHIFTS EQUIL.

\[ -ky_2 = mg \]

\[ F_{tot} = T_{sp} + T = -ky - mg \]

\[ y = 0 \quad \text{eq. of gravity} \]

\[ y' = y - y_g \]

\[ F_{tot} = -k(y' + y_g) - mg \]

\[ = -ky' - ky_g - mg \]

\[ F_{tot} = -ky' \quad \text{spring w/ grav from new & ps.} \]

Ch 11 - WORK

Work keeps track of energy transfer from a force applied over a distance.

Remember impulse \( J = \int F \, dt = \text{momentum transfer} \)

\( \Rightarrow \) work is energy analog

\[ \text{Signs:} \]

\[ \text{Work done on mass } m \]

\( \frac{\text{is positive}}{F \text{ and } dx} \)

\( \text{in same direction} \)

\( F = \text{force on } m \)
General case: \( \vec{F} \parallel \vec{d}r \)

- \( \vec{F} \) doesn't change speed \( \frac{d}{dt} \)
- So doesn't change \( K \)
- \( F_{||} \) changes speed \( \vec{d}r \) changes \( K \)

(Note - centripetal case does zero work!)

\[
F_{||} = ma_{||} = m \frac{d\vec{v}}{dt} = m \frac{d\vec{v}}{ds} \frac{ds}{dt} = m \frac{d\vec{v}}{ds}
\]

\[
\Delta K_{12} = \frac{1}{2} m \vec{v}_2^2 - \frac{1}{2} m \vec{v}_1^2 = \int_{s_1}^{s_2} m \vec{v} \cdot d\vec{v} = \int_{s_1}^{s_2} F_{||} ds = W_{12}
\]

Vector Dot Product

\[
F_{||} ds = F \cos \alpha \ ds
\]

\[
= \vec{F} \cdot d\vec{r} = \vec{F} \cdot dr
\]

Note: work is a transfer, not a state quantity like \( K \) or \( U \), so no \( d \) \* 

Note: circular motion \( \vec{F} \perp \vec{d}r \Rightarrow \vec{F} \cdot d\vec{r} = 0 \)

Spring:

\[
W = \int_{x_1}^{x_2} (kx)^2 dx = \frac{1}{2} kx^2
\]

\[
W = \frac{1}{2} k(x_2^2 - x_1^2)
\]

\( x > 0 \) as spring pushes object from \( x_1 \) to \( x_2 \)
Conservative Forces depend on position

and have potential energy \( W \) independent of path!

\[
W = -\Delta U = \int_{F} F \cdot dr = -\left( U(F_{2}) - U(F_{1}) \right)
\]

In general, if \( \vec{F} \) is conservative, then

\[
\vec{F} = -\nabla U(x)
\]

\[
= -\frac{du}{dx} \mathbf{i} - \frac{du}{dy} \mathbf{j} - \frac{dv}{dz} \mathbf{k}
\]

\( \Rightarrow \) common to work with \( U(F) \) rather than \( \vec{F} \)

<table>
<thead>
<tr>
<th>Conservative</th>
<th>Non-Conservative</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravity ( -mg ) (constant)</td>
<td>friction ( f ) depends on ( V )</td>
</tr>
<tr>
<td>Spring ( -kx )</td>
<td>(direction of motion)</td>
</tr>
<tr>
<td>Gravity ( -G \frac{mM}{r^{2}} )</td>
<td>air drag - depends on ( V )</td>
</tr>
</tbody>
</table>

 conserve energy in

"lose" mechanical

motion + potential

energy into heat

= mech. energy

"dissipative" force
Power = rate of doing work
\[ \frac{dw}{dt} = F \cdot dv \]

\[ p = \frac{dw}{dt} = F \cdot \frac{dv}{dt} = F \cdot v \]
\[ \frac{W}{s} = \frac{J}{s} \] (1 hp = 746 W)

Many other ways to calculate power, though

e.g. chairlift carries skiers up \( \frac{h}{300} \) m

rate = 1 chair / 5 s

chair = 2 people

eq avg. 150 kg w/gear

\[ p = \frac{mgh}{\Delta t} = \frac{150 \times 9.8 \times \frac{300}{5}}{5} = 90 \text{ kW} \]

Note that \( \frac{h}{\Delta t} \) is “speed” of m up hill

Terminal speed of car, air drag = wheel power

\[ F_d \delta = P \_w \] (30% loss in drive train

30% loss in drive train
doesn’t matter)

\[ \frac{1}{2} C_d A \frac{v^2}{\text{mph}} = 0.70 \text{ P_{eng}} \]

1 hp = 746 W = 0.75 kW

"scaling formula"

\[ U_{\text{max}} = \left( \frac{2 \cdot 0.70 \text{ P_{eng}}}{\rho AC_d} \right)^{1/3} \]

\[ U_{\text{max}} \approx 125 \text{ mph} \left( \frac{\text{P_{eng}}}{100 \text{ hp}} \right)^{1/3} = 56 \text{ m/s} = 125 \text{ mph} \]