Physics 202, Lecture 7

Today’s Topics

- Capacitance (Ch. 26)
  - Energy stored in capacitors
  - Dielectric and Capacitance

- Previews: Energy density, electric dipole moment, torques, dielectric material.
About Exam 1

- **When and where**
  - Monday Feb. 14\textsuperscript{th} 5:30-7:00 pm
  - (room to be announced)

- **Format**
  - Closed book
  - One 8x11 formula sheet allowed, **must be self prepared, no photo copying/download-printing of solutions, lecture slides, etc.**
  - 20-25 multiple choice questions
  - Bring a calculator (but no computer). Only basic calculation functionality can be used.
  - Bring a B2 pencil for Scantron.

- **Special requests:**
  - Should have been settled by now.
  - All specially arranged tests (e.g. those at alternative time) are held in our 202 labs. *(for approved requests only)*
Chapters Covered

- Chapter 23: Electric Fields
- Chapter 24: Gauss’s Law
- Chapter 25: Electric Potential
- Chapter 26: Capacitance

I will not post past/sample exams as none that I can find are representative. Often those can be misleading.

I will use next Thursday’s lecture to review for the test. (and will show a few sample test questions to help you get familiar with the test style)
Review: Capacitors and Capacitance

- A generic capacitor:

  \[ \Delta V \propto Q \Rightarrow Q = C \Delta V, \quad \Delta V = Q/C \]
Charging A Capacitor

Uncharged

Charging

Charged

\[ \Delta V = \frac{q}{C} \]

\[ \Delta V = \frac{Q}{C} \]

\[ du = \Delta V dq \]

Electric potential energy gained:

\[ U = \int du = \int \left( -\Delta V \right) (-dq) = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} \]

After charging the capacitor stores potential energy:

\[ U = \frac{1}{2} \frac{Q^2}{C} \]
Discharging A Capacitor

\[ \Delta V = \frac{Q}{C} \]

- \( Q \) \quad \text{Charged}
- \( -q \) \quad \text{dis-Charging}
- \( -Q \) \quad \text{Uncharged}

\[ dU = \Delta V dq \]

**Potential energy released:**

\[
U = \int dU = \int \Delta V (-dq) = \int_{Q}^{0} -\frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}
\]

\( \Rightarrow \) the originally charged capacitor has potential energy:

\[ U = \frac{1}{2} \frac{Q^2}{C} \]
Energy Stored In A Capacitor

- Show that energy required to charge a capacitor to a charge $Q$ is $U = \frac{1}{2} C(\Delta V)^2$

Solution: See previous two slides $\rightarrow$ energy stored in a capacitor is $U = \frac{1}{2} Q^2/C = \frac{1}{2} C(\Delta V)^2$

- Important: This energy is actually stored in the form of the electric field.

- Energy density of an Electric Field:
  
  \[ u_E = \frac{U}{\text{volume}} = \frac{U}{Ad} = \frac{1}{2} \frac{Q^2}{(CAd)} \]

  \[ = \frac{1}{2} \frac{Q^2}{(\varepsilon_0 A^2)} \]

  $\Rightarrow$ \[ u_E = \frac{1}{2} \varepsilon_0 E^2 \]
Quick Quiz

A parallel plate capacitor is holding a charge $q$. When the plates are pulled apart (i.e. $d$ increases), is the energy stored on it increasing or decreasing?

1. Increases
2. Decreases
3. Stays the same

Arguments:
- Volume increases
- While $E$ almost the same
- Work done to it when being pulling apart
Electric Dipole Moments

- Electric dipole moment $\mathbf{p}$.

\[
p = qd
\]

Note: $\mathbf{P}$ is a vector

- Electric dipole moment in constant $\mathbf{E}$ field

Net Force: $\sum \mathbf{F} = 0$

Net Torque: $\mathbf{\tau} = \mathbf{p} \times \mathbf{E}$

Potential energy: $U = -\mathbf{p} \cdot \mathbf{E}$
Quick Quiz

- Assuming a dipole moment can only rotate in an electric field, which of the following configurations is stable?

1. $45^\circ$

2. Points towards the E field

3. Points against the E field

4. Points normal to the E field
Dielectrics In External Field

- Dielectric material (dielectrics): Insulators that contain (microscopically) large amount of dipole moments
  - permanent
  - induced.
- Alignment of permanent dipoles in external field

Note: In an external field $E_0$, the induced field $E_{ind}$ is always opposite to $E_0$
Dielectrics And Capacitance

- When no dielectrics inserted
  \[ E = E_0 \implies \Delta V_0 = E_0 d \]
  \[ \text{capacitance } C_0 = \frac{Q}{\Delta V_0} \]

- After insertion of dielectrics
  \[ E_{\text{ind}} = - \alpha E_0 \]
  \[ \text{dielectric constant } \kappa > 1 \]

- \[ E = E_0 + E_{\text{ind}} = (1 - \alpha) E_0 = E_0 / \kappa \]

- \[ \Delta V = Ed = \frac{1}{\kappa} E_0 d = \frac{1}{\kappa} \Delta V_0 \]
  \[ \text{capacitance } C = \frac{Q}{\Delta V} = \kappa C_0 > C_0 \]

- Larger C
  - more Q stored per \( \Delta V \)
  - less \( \Delta V \) per fixed Q
  - More energy stored per given \( \Delta V \)
    (recall: \( U = \frac{1}{2} C \Delta V^2 \))
Use of Dielectric Material

- Often, a non-conducting dielectric material is inserted in-between conductor ends to increase capacitance.
Demo: Insert Dielectrics In Between Conductor Plates
Pre-Review Review

- Physics 201 reminder:
  Work, conservative force, and potential energy

- Work done by a force to an object:
  \[ W = \int \vec{F} \cdot d\vec{x} \]

- Work done by a conservative force is path independent:
  \[ W_{\text{conservative}} = \int_{A}^{B} \vec{F} \cdot d\vec{x} = -(U_B - U_A) \]

- Conservative work can be used to define potential energy.

- e.g.  \[ W_{\text{by gravity from A to B}} = mgh_A - mgh_B \]
  \[ \rightarrow U_B = mgh_B, U_A = mgh_A \rightarrow U = mgh \]
Quick Quizzes

☐ A person is lifting a block of weight. In the process
  ▪ Is the work done by the person positive or negative?
    • positive
  ▪ Is the work done by gravity positive or negative?
    • negative
  ▪ Is the gravitational potential energy of the weight increasing or decreasing?
    • increasing

☐ Back to Physics 202. Electric force is also a conservative force.
→ An electric potential energy can be defined.
→ In the process when the electric force is doing positive work, the electric potential energy decreases
→ In the process when the electric force is doing negative work, the electric potential energy increases