Physics 202, Lecture 4

Today’s Topics

- Conductors in Electrostatic Equilibrium (Ch. 24.4)
- Electric Potential (Ch. 25-Part I)
  - Electric Potential Energy & Electric Potential
  - Electric Potential And Electric Field

- Expected from preview:
  Conductors
  Conservative force, electric energy, electric potential difference, voltage….
Conductors And Electrostatic Equilibrium

- Conductors: In the absence of an external field, charges are evenly distributed throughout their volume. Negative charge (electrons) are able to move freely inside its body.
  - capable of redistributing charges when subject to an external electric field.
- Electrons will move around until their distribution is such that there is no force on them. Since \( F = qE = 0 \), \( E = 0 \) inside conductors.
- Charge redistribution \( \rightarrow \) eventually electrostatic equilibrium.

Initial \( \rightarrow \) transient, \( <10^{-16} \text{s} \) (right after \( E \) applied) \( \rightarrow \) equilibrium
Properties of Electrostatic Equilibrium

- Once in electrostatic equilibrium
  - The electric field is always zero inside the conductor.
  - E field on the surface of conductor is always normal (perpendicular) to the surface
    - if there were any parallel components, electrons would experience force and move
  - and has a magnitude of $\frac{\sigma}{\varepsilon_0}$ ($\sigma$ is the surface charge/area AT THAT POINT). (let’s derive this using Gauss’s law )
  - All net charges reside on the surface of conductor (i.e. no net charge inside the body of a conductor).

- And...
  - The electric field is also zero inside any cavity within the conductor. (why?)
  - Electric potential is the same over the whole conductor (Ch. 25)

The above properties are valid regardless of the shape and the total charge of the conductors!
Example

- Consider: solid insulating sphere of radius ‘a’ carrying charge ‘Q’ distributed uniformly throughout its volume. A conducting spherical shell concentric with the solid sphere has inner radius of ‘b’, outer radius of ‘c’, and carries charge -2Q.
  - Find the Electric field at 1, 2, 3, & 4.
Potential Energy (Phy201 Review)

- Ch-8: path independent work → conservative force.
- e.g. Gravitational Force is a conservative force (Ch-13):

\[ \vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12} \]

\[ W = \int_{path} \mathbf{F} \cdot d\mathbf{s} = \frac{G m_1 m_2}{r_f} - \frac{G m_1 m_2}{r_i} \]

Gravitational Potential energy:

\[ U = -\frac{G m_1 m_2}{r} \]

- Electric Force:

\[ \vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12} \]

\[ U = \frac{k_e q_1 q_2}{r} \]
Electric Potential Energy

Electric energy between two point charges:

\[ U = U - U_\infty = K_e \frac{q_0 q}{r} \]

- \( U \) is a scalar quantity
- \( U = 0 \) @ \( r = \infty \) (convenient convention)
- \( U \) can be positive or negative
  - +: between like-sign charges
  - -: between opposite charges
- SI unit: Joule (J)

Electric potential energy for system of multiple charges/charge distributions:

\[ U = \sum \text{of all combination of pairs.} \]

Integral if continuous distribution
Example: Three Charge system

- What is the work required to assemble the three charge system as shown? \((q_1=q_2=q_3=Q)\)
  Answer: \(k_e \frac{3Q^2}{a}\) (see board)

- Quiz: What if \(q_1=q_2=Q\) but \(q_3=-Q\)?
  Answer: \(-k_e \frac{Q^2}{a}\)
**Electric Potential Energy:**  
**Charge In An Electric Field**  
- Charge $q$ is subject an electric force in electric field $\mathbf{E}$.

\[ \mathbf{F} = q \mathbf{E} \]

- Work done by electric force:

\[ W = \int_{i}^{f} \mathbf{F} \cdot d\mathbf{s} = q \int_{i}^{f} \mathbf{E} \cdot d\mathbf{s} = -\Delta U \]

\[ \Delta U = U_f - U_i = -q \int_{i}^{f} \mathbf{E} \cdot d\mathbf{s} \]  

independent of $q$
Electric Potential Difference

- Electric Potential Energy: $q$ In a Generic E. Field

$$\Delta U = U_B - U_A = -q \int_A^B \mathbf{E} \cdot d\mathbf{s} = q\Delta V$$

- Electric Potential Difference

$$\Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = V_B - V_A$$
Properties of Electric Potential Difference

- It is defined upon the fact that the electric force is a conservative force.
- It is associated to the source field only and is independent of test charge.
- It has a unit: $J/C \equiv Volt (V)$
- It is commonly called as just Potential, but it is meaningful only as potential difference $V_B - V_A$.
- Usually a convenient point (remote, earth..) is chosen as “ground” $\rightarrow \Delta V = V - (V_A \equiv 0) = V$
- It is a scalar quantity. (No vector operation necessary!)
- $\Delta U = q \Delta V$
Exercise 1: Potential In Uniform E. Field

- In the uniform electric field shown.
  - Find E. potential at points: B,C,D,G
  - If a charge +q is placed at B,
  - what is the potential energy $U_B$? ($U_A = 0$)
    - If a charge –q is at B, what is $U_B$?
    - If a negative charge -q is initially at rest at G, will it move to A or B?
    - What is the kinetic energy when it reaches A?
Field lines always point towards lower electric potential.

Field lines and equal-potential lines are always at a normal angle.

In an electric field:
- A +q is always subject to a force in the same direction of field line. (i.e. towards lower V)
- A -q is always subject to a force in the opposite direction of field line. (i.e. towards higher V)

\[ F = qE \]
\[ \Delta U = q\Delta V \]
\[ W = \Delta U \]
\[ \Delta V = -\int E \, ds \]
Exercise 2: E. Potential and Point Charges

- In the configuration shown,
  - Find the potential difference $V_B - V_A$

**Answer:**

$$V_B - V_A = k_e \left( \frac{q}{r_B} - \frac{q}{r_A} \right)$$

(Exercise with your TA)
Exercise 3: Cathode Ray Tube (CRT)

Electrons are emitted with almost zero velocity on plate C, what is the energy per electron when they reach plate A? (Do with your TA)

\[ V_A - V_C = 12000 \text{V} \]
Visualization of Electric Potential
Equipotential Lines