## Review: Interference of Light Waves

- Two light waves with same color and amplitude.
  
  \[ E_1 = E_0 \sin(\omega t - kx_1 + \phi_1) \]
  
  \[ E_2 = E_0 \sin(\omega t - kx_2 + \phi_2) \]
  
  \[ E = E_1 + E_2 = 2E_0 \cos(\Delta \phi/2) \sin(\omega t + \phi) \]
  
  \[ \Rightarrow E_{\text{max}} = 2E_0 \cos(\Delta \phi/2) \]
  
  - Constructive interference: \( \Delta \phi = 0, 2\pi, 4\pi, \ldots \) \( E_{\text{max}} = 2E_0 \)
  
  - Destructive interference: \( \Delta \phi = \pi, 3\pi, 5\pi, \ldots \) \( E_{\text{max}} = 0 \)
  
  → It all depends on \( \Delta \phi \)!

### Path Length And Path Length Difference

For two interfering waves coming through different paths the phase difference:

\[ \Delta \phi = \Delta \phi_{\text{at the source}} + \Delta \phi_{\text{due to path}} + \Delta \phi_{\text{phase transition}} \]

\[ = 0 \text{ in many cases} = k(r_2 - r_1) = 2\pi/n(r_2 - r_1) \text{ see later} \]

where \( r_1 \) and \( r_2 \) are path lengths, \( \Delta r = (r_2 - r_1) \) is called path length difference.
Thin Film Interference

- Thin film splits light → split lights then interfere
- Lights 1, 2 interfere
- Phase change $\pi$ for light 1
- $\Delta \phi_{12} = 2\pi/\lambda_n (2t) + \pi$

Quiz:
Constructive/destructive Conditions?

Exercise: Non Reflective Coating

- Determine the minimum thickness (t) of SiO coating so a light of 550nm is non-reflective at the surface.
- Solution (see board):
  - Non “reflective” → 1 and 2 cancel each other (destructive interference)
  - $\Delta \phi_{12} = 2\pi/\lambda_n 2t + 0^\circ = \pi$
  - $t = \lambda_n/4 = \lambda/(4n) = 94.8$ nm.
  - Note t is $\lambda$ dependent.

Exercise: Pro-Reflective Coating

- Determine the minimum thickness (t) of SiO coating so a light of 550nm is max-reflective at the surface.
- Solution (see board):
  - Pro- “reflective” → 1 and 2 interference constructively
  - $\Delta \phi_{12} = 2\pi/\lambda_n 2t + 0^\circ = 0$, or $2\pi$
  - $t = \lambda_n/2 = \lambda/(2n) = 189.6$ nm.
  - Note t is $\lambda$ dependent.
Color Separation: Make Colorful World Out Of (White) Daylight

- Color: Light with certain frequency.
- Daylight: a mixture of all colors → appears white.
- Three ways to make daylight colorful:
  - Filtering: Only one color is allowed to pass
  - Dispersion: Different colors at different refractive angles
  - Interference: Different colors get enhanced/weakened at different path-length difference, which is a function of thickness, observing angle, etc. (pictures next page)

Condition for Ray Approximation

- When the wavelength of the light is much smaller than the size of the optical objects it encounters, it can be treated as (colored) rays.

Ray approximation is valid when \( \lambda << d \)

Ray approximation is not valid near the gap when \( \lambda \approx d \). OK elsewhere

Colorful Interference Patterns

Single-Slit Interference (Single-Slit Diffraction)

If lights were just rays
Single-Slit Diffraction Pattern Explained

The slit is not a point source → Interference

\[ E_y = \sum E_i \]
\[ = \sum \left( \frac{\Delta y}{D} E_0 \right) \sin(\omega t + 2\pi \frac{\lambda}{D} \sin \theta) \]
\[ = \int_{-D/2}^{D/2} E_0 \sin(\omega t + 2\pi \frac{\lambda}{D} \sin \theta) dy \]
\[ = \frac{2E_0}{D} \int \sin(2\pi \frac{\lambda}{D} \sin \theta) \sin(\omega t) dy \]
\[ = 2E_0 \frac{\Delta y}{D} \sin \theta \sin(\omega t) \]

The text also offers a derivation using phasors.

Where Are the Dark Fringes?

The dark fringes occur at:

\[ I = I_0 \sin(\beta/2) \]
\[ \beta = \frac{2\pi}{\lambda} D \sin \theta \]

\[ \sin(\beta/2) = 0 \]
\[ \sin \theta_{\text{dark}} = m \lambda / D, \ m = \pm 1, \pm 2, \pm 3, \ldots \]

\[ \Delta \theta = \frac{2\lambda}{D} \]

Central bright dot width \( \Delta \theta = \frac{2\lambda}{D} \)

First dark fringes at = \( \pm \frac{\lambda}{D} \)