Today’s Topics

- AC Circuits with AC Source
- AC Power Source
- Resistors, Capacitors and Inductors in AC Circuit
- RLC Series in AC Circuit
- Impedance
- Resonances in Series RLC Circuit
- Power in AC Circuit
- Transformers

AC Power Source

\[ \Delta V = \Delta v_{\text{max}} \sin(\omega t + \phi_0) = \Delta v_{\text{max}} \sin(\omega t) \]

Initial phase at \( t = 0 \)

(usually set \( \phi_0 = 0 \))

\( \omega = 2\pi f \)

\( T = \frac{2\pi}{\omega} \)

Recap

- A sinusoidal function \( x = A \sin \phi \) can be represented graphically as a phasor vector with length \( A \) and angle \( \phi \) (w.r.t. to horizontal)

Phasor

Resistors in an AC Circuit

\[ \Delta V - R = 0 \text{ at any time} \]

\[ i_R = \frac{\Delta V}{R} = I_{\text{max}} \sin \omega t \]

The current through an resistor is in phase with the voltage across it
**Inductors in an AC Circuit**

- \( \Delta V - L \frac{di}{dt} = 0 \)

- \( i_L = i_{max} \sin(\omega t - \pi/2) \)
- \( i_{max} = \frac{\Delta V_{max}}{\omega L} = \frac{\Delta V_{max}}{X_L} \)
- \( X_L = \omega L \rightarrow \text{inductive reactance} \)
- The current through an inductor is 90\(^\circ\) behind the voltage across it.

**Capacitors in an AC Circuit**

- \( \Delta V - \frac{q}{C} = 0, \frac{dq}{dt} = i \)

- \( i_C = i_{max} \sin(\omega t + \pi/2) \)
- \( i_{max} = \frac{\Delta V_{max}}{1/(\omega C)} = \frac{\Delta V_{max}}{X_C} \)
- \( X_C = 1/(\omega C) \rightarrow \text{capacitive reactance} \)
- The current through a capacitor is 90\(^\circ\) ahead of the voltage across it.

**Summary of Phasor Relationship**

- \( I_R \) and \( \Delta V_R \) in phase
- \( |i_R| = |\Delta V_R|/R \)
- \( I_L \) 90\(^\circ\) behind \( \Delta V_L \)
- \( |i_L| = |\Delta V_L|/X_L \)
- \( I_C \) 90\(^\circ\) ahead of \( \Delta V_C \)
- \( |i_C| = |\Delta V_C|/X_C \)

**AC Circuit: Series RLC**

- Find out current \( i \) and voltage difference \( \Delta V_R, \Delta V_L, \Delta V_C \).

**Notes:**
- Kirchhoff’s rules still apply!
- Phasor analysis is convenient.
AC Circuit: Series RLC

- Find out current $i$ and voltage difference $\Delta V_R$, $\Delta V_L$, $\Delta V_C$.

**Things we know:**
1. Current everywhere must be the same $i = I_{max} \sin(\omega t)$
2. $\Delta V_R$ has the same phase as current
3. $\Delta V_L$ leads the current by 90°
4. $\Delta V_C$ lags the current by 90°

$\Delta V_R = I_{max} R \sin(\omega t)$

$\Delta V_L = I_{max} X_L \sin(\omega t + \pi/2)$

$\Delta V_C = I_{max} X_C \sin(\omega t - \pi/2)$

Note: Phase relations on p.962 for $(\Delta v$ and $i)$ to $(\Delta V_R$, $\Delta V_L$ and $\Delta V_C)$ are wrong in the book.

**Phasor Technique**
- The phasor of $\Delta v_{RLC}$ = vector sum of phasors for $\Delta V_R$, $\Delta V_L$, $\Delta V_C$.

$\Delta V_{max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2}$

$= I_{max} \sqrt{R^2 + (X_L - X_C)^2}$

$\phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$

note: $X_L = \omega L$, $X_C = \frac{1}{\omega C}$

$\Delta V = \Delta V_{max} \sin(\omega t + \phi)$

$i = I_{max} \sin(\omega t)$
Resonances In Series RLC Circuit

- The impedance of an AC circuit is a function of \( \omega \).
  - e.g Series RLC: \( Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \frac{1}{\omega C}} \)
- when \( \omega = \omega_0 = \frac{1}{\sqrt{LC}} \) (i.e. \( X_L = X_C \))
  - lowest impedance \( \Rightarrow \) largest current \( \Rightarrow \) resonance

For a general AC circuit, at resonance:
- Impedance is at lowest
- Phase angle is zero. (I is “in phase” with \( \Delta V \))
- \( I_{max} \) is at highest
- Power consumption is at highest

Demo: [http://ngsir.netfirms.com/englishhtm/RLC.htm](http://ngsir.netfirms.com/englishhtm/RLC.htm)

Summary of Impedances and Phases

<table>
<thead>
<tr>
<th>Circuit Elements</th>
<th>Impedance ( Z )</th>
<th>Phase Angle ( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>( R )</td>
<td>0°</td>
</tr>
<tr>
<td>( X_L )</td>
<td>( X_L )</td>
<td>90°</td>
</tr>
<tr>
<td>( X_C )</td>
<td>( -X_C )</td>
<td>-90°</td>
</tr>
<tr>
<td>( X_C )</td>
<td>( X_L )</td>
<td>90°</td>
</tr>
<tr>
<td>( R )</td>
<td>( R + \frac{1}{\omega L} )</td>
<td>Negative, between (-90°) and (0°)</td>
</tr>
<tr>
<td>( X_L )</td>
<td>( \sqrt{R^2 + X_C^2} )</td>
<td>Positive, between (0°) and (90°)</td>
</tr>
</tbody>
</table>
| \( X_C \)        | \( \sqrt{R^2 + (X_L - X_C)^2} \) | Negative if \( X_C > X_L \)
  - Positive if \( X_C < X_L \) |

* In each case, an AC voltage (not current) is applied across the elements.

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Comparison Between Impedance and Resistance

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Resistance</th>
<th>Impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>R</td>
<td>Z</td>
</tr>
<tr>
<td>Application</td>
<td>Circuits with only R</td>
<td>Circuits with R, L, C</td>
</tr>
<tr>
<td>Value Type</td>
<td>Real</td>
<td>Complex: ( Z=</td>
</tr>
<tr>
<td>( \Delta V ) Relationship</td>
<td>( \Delta V=IR )</td>
<td>( \Delta V=IZ, \Delta V_{max}=</td>
</tr>
<tr>
<td>In Series:</td>
<td>( R=R_1+R_2+R_3+\ldots )</td>
<td>( Z=Z_1+Z_2+Z_3+\ldots )</td>
</tr>
<tr>
<td>In Parallel:</td>
<td>( \frac{1}{Z}=\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}+\ldots )</td>
<td>( I=I_1+I_2+I_3+\ldots )</td>
</tr>
</tbody>
</table>

Power in AC Circuit

- Power in a circuit: \( P(t) = i(t)\Delta V(t) \) true for any circuit, AC or DC
- In an AC circuit, current and voltage on any component can be written in general:
  - \( \Delta V(t) = \Delta V_{max} \sin(\omega t) \)
  - \( i(t) = I_{max} \sin(\omega t - \phi) \)

\[
P(t) = I_{max} \sin(\omega t - \phi) \times \Delta V_{max} \sin(\omega t) \\
= I_{max} \Delta V_{max} \sin(\omega t - \phi) \sin(\omega t) \\
P_{average} = \frac{1}{2} I_{max} \Delta V_{max} \sin(\omega t - \phi) \sin(\omega t) \quad \text{(see board)}
\]
Power in AC Circuit

For resistor: $\phi = 0$

$P_{\text{average}} = \frac{1}{2} I_{\text{max}} \cdot \Delta V_{\text{max}}$

For inductor: $\phi = \pi/2$

$P_{\text{average}} = \frac{1}{2} I_{\text{max}} \cdot \Delta V_{\text{max}} \cdot \cos(\pi/2) = 0$

For Capacitor: $\phi = -\pi/2$

$P_{\text{average}} = \frac{1}{2} I_{\text{max}} \cdot \Delta V_{\text{max}} \cdot \cos(\pi/2) = 0$

(Ideal inductors and capacitors NEVER consume energy!)

For a general AC circuit:

$P_{\text{ave}} = \frac{1}{2} I_{\text{max}} \cdot \Delta V_{\text{max}} \cdot \cos(\phi) = \frac{1}{2} I_{\text{max}}^2 R = \frac{1}{2} (\Delta V_{\text{max}})^2 / R$

$= I_{\text{rms}}^2 R = \Delta V_{\text{rms}}^2 / R$

- Commonly used format:

$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}}$, $\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}}$

Transformers

- Useful for converting AC voltages
  - Power distribution
  - Charger for cell phone, laptop, etc. etc...

Voltage across primary:

$\frac{V_p}{N_p} = \frac{d\Phi_p}{dt}$

Voltage across secondary:

$\frac{V_s}{N_s} = -\frac{d\Phi_s}{dt}$

Did you ever wonder...

- why/how the cell phone chargers have gotten so much lighter and smaller?

10 years ago

5 years ago

Now