Physics 202, Lecture 12

Today’s Topics

- Magnetic Field (Ch 29, part II)
- Motion of a Charged Particle In a Uniform \( \mathbf{B} \) Field
- Applications:
  - Magnetic Confinement
  - First Modern Particle Accelerator: Cyclotron
  - Mass Selector (\( \frac{q}{m} \))
- Force Between Two Current Carrying Wires
- Force and Torque on a Current Loop In Uniform \( \mathbf{B} \) Field
- Magnetic dipoles (will be reviewed next lecture)
- Example: Hall Effect (will be reviewed next lecture)

Properties of Magnetic Force

- Magnetic Force: \( \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \).  \( \mathbf{F}_B = |q| \mathbf{v} \mathbf{B} \sin \theta \)
  - \( \mathbf{F}_B = 0 \) if \( \mathbf{v} = 0 \)
  - \( \mathbf{F}_B = 0 \) if \( \mathbf{v} \) and \( \mathbf{B} \) in \( 0^\circ \) or \( 180^\circ \)
  - \( \mathbf{F}_B \) is normal to \( \mathbf{v} \)
  - \( \mathbf{F}_B \) is normal to \( \mathbf{B} \)
  - work done by \( \mathbf{F}_B \) is always zero! (force is perpendicular to the displacement)
  - direction of \( \mathbf{F}_B \) are opposite for positive charge and negative charges.

Motion Of Charged Particle in a Uniform \( \mathbf{B} \) Field

- Exercise: Show that if a charged particle \( q \) of mass \( m \) in a uniform \( \mathbf{B} \) field has an initial velocity \( \mathbf{v} \) in the plane perpendicular to \( \mathbf{B} \), its motion is a uniform circular motion in that plane with
  - radius \( r = \frac{mv}{qB} \)
  - period:
    \( T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \)
  - Note: \( T \) is independent of \( v \)

(recall: uniform circular motion)
**Application: Magnetic Confinement**

- Tokamak
- Magnetic Bottle

**MST: Madison Symmetric Torus**

**Application: Cyclotron**

*(First Modern Charged Particle Accelerator)*

- First Cyclotron (1934)
- Lawrence & Livingston

**Application: Mass Selector**

- Speed selected: $v = \frac{E}{B}$
- Mass selected: $\frac{m}{q} = \frac{rB}{(E/B)}$
Mass spectrometry examples

- Analysis of proteins and peptides
- Volcanic ash content in Antarctic Ice

Mass Selector: J.J Thomson Apparatus (1897)
- This is your lab next week: measuring e/m

Magnetic Force On Current Carrying Wire Segment
- Magnetic force on a current segment of length $L$ in uniform field $B$:
  $$F_B = 2qv_x B = I L x B$$
- Current: moving charges. $I = qnv_x A$
- Magnetic force on charge $q$: $qv_x B$
- $F_B = qv_x x B$ (ALn) = $I L x B$

Magnetic Force On A Current Carrying Wire

- Top View

Also demo: Bending electron beam
Magnetic Force On Current Carrying Wire

- Magnetic force on a curved wire in uniform field \( B \): \( \mathbf{F} = I \mathbf{L} \times \mathbf{B} \)

Note: Net force on a current loop in uniform \( B \) field is zero

Review Exercise: Forces On A Current Loop

- For a current loop in a uniform magnetic field as shown, what is the direction of the force on each side?

Torque on a Current Loop In Uniform \( B \) Field

- Exercise: For a current loop in a uniform \( B \) field, show that the torque on the loop is: \( \vec{\tau} = I \mathbf{A} \times \mathbf{B} \)

Conveniently, the result can be rewritten as:

Review: Electric Dipole Moments

- Electric dipole moment \( \mathbf{p} \):

\[ \mathbf{p} = q \mathbf{d} \]

\[ \sum \mathbf{F} = 0 \]

\[ \vec{\tau} = \mathbf{p} \times \mathbf{E} \]

\[ U = -\mathbf{p} \cdot \mathbf{E} \]
Magnetic Dipole Moments

- Magnetic dipole moment $\mu$.

**Macroscopic**

$\mu = I A$

**Microscopic**

$\mu \propto L$

angular momentum of orbiting or spin

definition of magnetic moment

\[ \sum F = 0 \]
\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
\[ U = -\vec{\mu} \cdot \vec{B} \]

$\mu$ in B Field