Physics 202, Lecture 10

Today’s Topics

- Direct Current Circuits (Ch. 28)
- Basic circuit components (ε, R, …)
- Kirchhoff’s Rules
- Circuits Analysis (For circuits of R’s and ε’s)

- Preview Requirements: emf, junction rule, loop rule, ….

Next Lecture:
RC circuit (ch. 28.4), Magnetic Field (ch. 29.1-3)

Exam 1 Results

Median: 75%
Mean: 72%

Very rough curving: A ~ 85% up, bottom B=75%, D=50% and below

Basic Circuit Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbol</th>
<th>Behavior in circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal battery, emf</td>
<td>ε</td>
<td>ΔV=V+ - V- =ε</td>
</tr>
<tr>
<td>Resistor</td>
<td>R</td>
<td>ΔV = IR</td>
</tr>
<tr>
<td>Realistic Battery</td>
<td>ε, R</td>
<td>ΔV = ε, R = ε</td>
</tr>
<tr>
<td>(Ideal) wire</td>
<td></td>
<td>ΔV = 0 (⇒ R = 0, L = 0, C = 0)</td>
</tr>
<tr>
<td>Capacitor</td>
<td></td>
<td>ΔV = V, -V, q/C, dq/dt = I</td>
</tr>
<tr>
<td>Inductor</td>
<td></td>
<td>ΔV = - LdI/dt</td>
</tr>
<tr>
<td>(Ideal) Switch</td>
<td></td>
<td>L = 0, C = 0, R = 0 (on), R = ∞ (off)</td>
</tr>
<tr>
<td>Transformer</td>
<td></td>
<td>Future Topics</td>
</tr>
<tr>
<td>Diodes, Transistors,</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Simple Circuit 1: Resistors In Series

- Exercise: show Req = R1 + R2.
  - $I_1 = I_2 = I$
  - $ΔV = V_{R1} + V_{R2}$
  - $= IR_1 + IR_2$
  - $⇒ ΔV = I(R_1 + R_2)$
  - i.e. $R_{eq} = R_1 + R_2$

- In general: $R_{eq} = R_1 + R_2 + R_3 + ….$
Simple Circuit 2: Resistors In Parallel

- Show \( 1/R_{eq} = 1/R_1 + 1/R_2 \).
  - \( V_{in} = V_{out} = \Delta V \)
  - \( I_1 + I_2 = I \)
  - \( \Delta V (1/R_1 + 1/R_2) = I \)
  - \[ \Delta V = I (1/(1/R_1 + 1/R_2)) \]

- In general: \( 1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots \)

Quiz/Exercise: Equivalent Resistance of a Combined Parallel and Serial Circuit

- What is the \( R_{eq} \) for the combination shown?
  - \( R_1 = R_2 = 1\Omega, R_3 = 2\Omega, R_4 = 4\Omega \).
  - 1. 8Ω
  - 2. 6Ω
  - 3. 5Ω
  - 4. None of above

A Complicated Circuit

A complicated circuit:
- May contain more than one emf
- May not be simplified as “in series” or “in parallel”
- May contain multi loops and junctions.

Kirchhoff’s Rules: Junction Rule

- Junction Rule (Charge conservation):
  - The sum of currents entering any junction equals the sum of currents leaving that junction.
  - \( \sum I_{in} = \sum I_{out} \)

- In practice, the classifications of “in” and “out” are determined by assigned direction for each current.
  - “in”: current with assigned direction towards junction
  - “out”: current with assigned direction off junction
(Very) Quick Quiz: Junction Rule

What is the junction rule for the current assignment shown?

1. \( I_1 + I_2 = I_3 \)
2. \( I_1 - I_2 = I_3 \)
3. Neither

Although equation 2 and 3 are equivalent, equation 3 does not follow template form \( I_{in} = I_{out} \).

Quick Quiz: Junction Rule

What is the junction rule for the current assignment shown?

1. \( I_1 + I_2 = I_3 \)
2. \( I_1 + I_2 + I_3 = 0 \)
3. Neither

While the actual currents can not all go into a junction, the assigned currents can.

Kirchhoff’s Rules: Loop Rule

- Loop Rule (Energy Conservation):
  The sum of potential drops across components along any closed circuit loop must be zero.
  \[ \sum \Delta V = 0 \]

- The potential “drop” across a component is always defined as \( V_{down\_stream\_end} - V_{up\_stream\_end} \) where, the stream direction is the same as the loop direction.

- The exact expression of the potential drop is determined by the type of component and the assigned current direction. (See next slides)

Determine Potential Difference

- For a resistor:
  \[ \Delta V = V_a - V_b = -IR \]

- For a voltage source:
  \[ \Delta V = V_a - V_b = +ε \]
**Steps to Apply Kirchhoff’s Rules**

1. Assign a directional current for each branch (segment) of a circuit. The assigned direction for each current can be arbitrarily chosen but, once assigned, need to be observed.

2. Set up junction rules at certain (any) junctions. Normally, # of junctions = # of branches -1.

3. Select a number of closed loops to apply loop rule. For each closed loop, assign a loop direction (clockwise or counter clockwise). Follow that assigned direction, find ΔV drop across each component, and apply loop rule. # of loops determined by # of unknowns.

4. Solve for unknowns.

5. If a current is found to be negative, it means its actual direction is opposite to the originally chosen one. The magnitude is always correct.

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**Example 1: Multi-Loop**

- Text example 28.7
- Find out $I_1$, $I_2$, $I_3$

Kirchhoff’s Rules:

- **Junction c:**
  \[ I_1 + I_2 = I_3 \]

  - **Loop abcda:**
  \[ -\varepsilon_1 + I_1 R_1 - \varepsilon_1 - I_3 R_3 = 0 \]

  - **Loop befcb:**
  \[ -\varepsilon_2 + I_1 R_1 - \varepsilon_2 - I_2 R_2 = 0 \]

  Solving three equations:
  \[ I_1 = 2.0A, I_2 = -3.0A, I_3 = -1.0A, \]
  What does the – sign mean?

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**Example 1 Again: Different Initial Directions**

Different initial direction for $I_1$, $I_2$

- Apply Kirchhoff’s Rules:

  - **Junction c:**
  \[ 0 = I_3 + H_1 + H_2 \]

  - **Loop abcda:**
  \[ \varepsilon_1 + I_1 R_1 - I_3 R_3 = 0 \]

  - **Loop befcb:**
  \[ -\varepsilon_2 - I_1 R_1 - \varepsilon_2 + I_2 R_2 = 0 \]

  Solving three equations:
  \[ I_1 = -2.0A, I_2 = +3.0A, I_3 = -1.0A, \]
  Same effective result as in previous slide