Today’s Topics

- Direct Current Circuits (Ch. 28)
- Basic circuit components (ε, R, …)
- Kirchhoff’s Rules
- Circuits Analysis (For circuits of R’s and ε’s)

Preview Requirements:
emf, junction rule, loop rule, ….

Next Lecture:
RC circuit (ch. 28.4), Magnetic Field (ch. 29.1-3)
Exam 1 Results

Very rough curving: A ~ 85% up, bottom B=75%, D=50% and below

Median: 75%
Mean: 72%
# Basic Circuit Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbol</th>
<th>Behavior in circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal battery, emf</td>
<td><img src="ideal_battery.png" alt="Symbol" /></td>
<td>$\Delta V = V_+ - V_- = \varepsilon$</td>
</tr>
<tr>
<td>Resistor</td>
<td><img src="resistor.png" alt="Symbol" /></td>
<td>$\Delta V = -IR$</td>
</tr>
<tr>
<td>Realistic Battery</td>
<td><img src="realistic_battery.png" alt="Symbol" /></td>
<td>$\Delta V = 0$ ($\Rightarrow R=0, L=0, C=0$)</td>
</tr>
<tr>
<td>(Ideal) wire</td>
<td><img src="ideal_wire.png" alt="Symbol" /></td>
<td>$\Delta V = V_- - V_+ = -q/C$, $dq/dt = I$</td>
</tr>
<tr>
<td>Capacitor</td>
<td><img src="capacitor.png" alt="Symbol" /></td>
<td>$\Delta V = -LdI/dt$</td>
</tr>
<tr>
<td>Inductor</td>
<td><img src="inductor.png" alt="Symbol" /></td>
<td>$\Delta V = -LdI/dt$</td>
</tr>
<tr>
<td>(Ideal) Switch</td>
<td><img src="ideal_switch.png" alt="Symbol" /></td>
<td>$L=0$, $C=0$, $R=0$ (on), $R=\infty$ (off)</td>
</tr>
<tr>
<td>Transformer</td>
<td><img src="transformer.png" alt="Symbol" /></td>
<td>Future Topics</td>
</tr>
<tr>
<td>Diodes, Transistors,...</td>
<td><img src="diodes_transistors.png" alt="Symbol" /></td>
<td>Future Topics</td>
</tr>
</tbody>
</table>
Simple Circuit 1: Resistors In Series

- Exercise: show $R_{eq} = R_1 + R_2$.
  - $I_1 = I_2 = I$
  - $\Delta V = V_{R1} + V_{R2}$
    $= IR_1 + IR_2$
    $\Rightarrow \Delta V = I(R_1 + R_2)$
  - i.e. $R_{eq} = R_1 + R_2$

- In general: $R_{eq} = R_1 + R_2 + R_3 + \ldots$. 
Simple Circuit 2: Resistors In Parallel

- Show $1/R_{eq}=1/R_1 + 1/R_2$.
  - $V_{R1}=V_{R2} = \Delta V$
  - $I_1 + I_2 = I$
  - $I_1 R_1 /R_1 + I_2 R_2 /R_2 = I$
  - $\Delta V (1/R_1 + 1/R_2) = I$
  - $\Delta V = I \left( \frac{1}{(1/R_1 + 1/R_2)} \right)$

  i.e.

  $1/R_{eq} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$

- In general: $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$
Quiz/Exercise: Equivalent Resistance of a Combined Parallel and Serial Circuit

What is the $R_{eq}$ for the combination shown? $R_1 = R_2 = 1\,\Omega$, $R_3 = 2\,\Omega$, $R_4 = 4\,\Omega$.

1. 8\,\Omega
2. 6\,\Omega
3. 5\,\Omega
4. None of above
A complicated circuit:

- May contain more than one emf
- May not be simplified as “in series” or “in parallel”
- May contain multi loops and junctions.
Kirchhoff’s Rules: Junction Rule

- Junction Rule (Charge conservation):
  The sum of currents entering any junction equals the sum of currents leaving that junction.

\[ \Sigma I_{\text{in}} = \Sigma I_{\text{out}} \]

- In practice, the classifications of “in” and “out” are determined by assigned direction for each current.
  The assignment of current directions can be arbitrary. They may not be the same as actual directions, which are not known a priori.
  - “in” : current with assigned direction towards junction
  - “out” : current with assigned direction off junction
What is the junction rule for the current assignment shown?

1. \( I_1 + I_2 = I_3 \)
2. \( I_1 = I_2 + I_3 \)  
3. \( I_1 - I_2 = I_3 \)

Although equation 2 and 3 are equivalent, equation 3 does not follow template form \( I_{in} = I_{out} \).
Quick Quiz: Junction Rule

What is the junction rule for the current assignment shown?

1. $I_1 + I_2 = I_3$
2. $I_1 + I_2 + I_3 = 0$
3. Neither

While the actual currents cannot all go into a junction, the assigned currents can.
Kirchhoff’s Rules: Loop Rule

- Loop Rule (Energy Conservation):
The sum of potential drops across components along any closed circuit loop must be zero.

\[ \Sigma \Delta V = 0 \]

- The potential “drop” across a component is always defined as
  \( V_{\text{down\_stream\_end}} - V_{\text{up\_stream\_end}} \)
  where,
  the stream direction is the same as loop direction

- The exact expression of the potential drop is determined by the type of component and the assigned current direction. (See next slides)
Determine Potential Difference

\[ \Delta V = V_b - V_a = -\varepsilon \]

\[ \Delta V = V_b - V_a = +\varepsilon \]

\[ \Delta V = V_b - V_a = -IR \]

\[ \Delta V = V_b - V_a = +IR \]
Steps to Apply Kirchhoff’s Rules

1. Assign a directional current for each branch (segment) of a circuit. The assigned direction for each current can be arbitrarily chosen but, once assigned, need to be observed.

2. Set up junction rules at certain (any) junctions. Normally, # of junctions = # of branches -1.

3. Select a number of closed loops to apply loop rule. For each closed loop, assign a loop direction (clockwise or counter clockwise). Follow that assigned direction, find ΔV drop across each component, and apply loop rule. # of loops determined by # of unknowns.

4. Solve for unknowns.

5. If a current is found to be negative, it means its actual direction is opposite to the originally chosen one. The magnitude is always correct.
Example 1: Multi-Loop

(Text example 28.7)

Find out $I_1$, $I_2$, $I_3$

Kirchhoff’s Rules:

Junction c:

$\sum I_1 + I_2 = I_3$

Loop abcd:

$\varepsilon_1 - I_1R_1 - I_3R_3 = 0$

Loop befcb:

$-\varepsilon_2 + I_1R_1 - \varepsilon_1 - I_2R_2 = 0$

Solving three equations:

$I_1 = 2.0\, \text{A}$, $I_2 = -3.0\, \text{A}$, $I_3 = -1.0\, \text{A}$,

What does the – sign mean?
Example 1: Interpretation of Results

\[ I_1 = 2.0\text{A}, \quad I_2 = -3.0\text{A}, \quad I_3 = -1.0\text{A}, \]

Actual situation
Example 1 Again: Different Initial Directions

Different initial direction for $I_1$, $I_2$

- Apply Kirchhoff’s Rules:

  Junction c:
  $0 = I_3 + I_1 + I_2$

  Loop abcda:
  $\varepsilon_1 + I_1 R_1 - I_3 R_3 = 0$

  Loop befcb:
  $-\varepsilon_2 - I_1 R_1 - \varepsilon_1 + I_2 R_2 = 0$

Solving three equations:

$I_1 = -2.0 \text{A}$, $I_2 = +3.0 \text{A}$, $I_3 = -1.0 \text{A}$, 

Same effective result as in previous slide