Chapter 5, Lect. 9
Additional Applications of Newton’s Laws
Today: Fractional forces, drag forces

- When and where: About Midterm Exam 1
  - Thurs. Feb. 17th 5:45-7:00 pm
  - Rooms: See course webpage. Be sure to report to your TA’s room
  - Your TA will give a review during the discussion session next week.
- Format
  - Closed book, 20 multiple-choice questions (consult with practice exam)
  - 1 page 8x11 formula sheet allowed, must be self-prepared, no photo copying/download-printing of solutions, lecture slides, etc.
  - Bring a calculator (but no computer). Only basic calculation functionality can be used. Bring a #2 pencil for Scantron.
  - Fill in your ID and section #!
- Special requests:
  - One alternative exam all set:
    - 3:30pm – 4:45pm, Thurs Feb 17th, room 5280 Chamberlin (I).

Frictional Force

- Friction
  - Opposes motion between systems in contact
  - Parallel to the contact surface
  - Depends on the force holding the surfaces together
    \[ \Rightarrow \text{the normal force } N \]
  - and a constant: coefficient of friction \( \mu \)
- Static friction
  - Frictional force without relative motion
    \[ f_s \leq \mu s N \]
- Kinetic friction
  - Frictional force on an object in motion
    \[ f_k = \mu k N \]

Surface friction...

- Force of friction acts to oppose relative motion:
  - Parallel to surface.
  - Perpendicular to normal force
  - It is determined (proportional to) normal force.

Application: ABS (Anti-lock Brake System)

- Anti-lock brakes work by making sure the wheels roll without slipping. This maximizes the frictional force slowing the car since \( \mu_s > \mu_k \).
Initial speed $v_0$: Find stopping distance

Normal force is balanced by gravity because there is no vertical motion: $N = Mg$, if $M$ is the mass of the object.

Kinetic frictional force that decelerates the block is, $f = \mu_k N = \mu_k Mg$.

Therefore, deceleration (direction opposite of $v_0$), $a = -\frac{f}{M} = -\frac{\mu_k N}{M} = -\frac{\mu_k Mg}{M} = -\mu_k g$.

Given deceleration, use kinematics equation to obtain the answer:

$$s = \frac{v^2}{2a} = \frac{v_0^2}{2(-\mu_k g)} = \frac{v_0^2}{2\mu_k g}$$

Answer: $16.3$ m

$\text{-- independent of the mass}$

$\text{-- the smaller } \mu_k \text{ is, the larger the stopping distance will be.}$

---

Example 2

Find acceleration of the block $M$.

Draw free body diagram.

Resolve $T$ in $x$ and $y$ components: $T_x = T \sin \theta$, $T_y = T \cos \theta$.

Solve for $y$-component of force: $N + T_y = Mg$.

Solve for $x$-component of force: $F_x = T_x - F_s$, with $F_s = \mu_k N$.

Then use $a = \frac{F_x}{M}$.

$$a = \frac{T \sin \theta - \mu_k N}{M} = \frac{T M \sin \theta - \mu_k M g}{M^2} = \frac{T \sin \theta - \mu_k g}{M}$$

Answer: acceleration of block is $6.0 \text{ m/s}^2$ in $+x$ direction.

---

Example 3:

- Consider $M$ on an inclined plane, with an angle $\theta$, w.r.t the horizontal.

$\text{-- In this case, the force provided by friction will depend on the angle } \theta \text{ of the plane: because of the normal force.}$
Consider i and j components of $F_{\text{NET}} = ma$

i component: $mg \sin \theta - \mu N = ma$

j component: $N = mg \cos \theta$

$m g \sin \theta - \mu N g \cos \theta = ma$

$a = \sin \theta - \mu \cos \theta$

If $a = 0$ → Static friction!

As long as $\mu > \tan \theta$

Is the frictional force always opposite to the moving direction?

- Friction keeps the car wheels from spinning in place
- You want the tires to roll, clockwise to your view.
- Friction opposes it
- The contact point is at rest - although the car is in motion

What matters is the coefficient of static friction!

Consider Newton's 3rd law:

$F_{\text{road on car}}$ is the actual force ON the car.

Static Friction $N$ is its maximum value

A kid on a toboggan

Naively, Child: $f_c = m_c a$

Toboggan (slippery ground): $F - f_c = m_c a$

Thus: $F = m_c a + m_i a \rightarrow a = F/(m_c + m_i)$

However, the static frictional force can't help forever!

Then the maximum acceleration: $m_c a_{\text{max}} = f_{c,\text{max}}$

Thus: $a = F/(m_c + m_i) \leq \mu_c g$

Drag Forces

- Objects moving through a fluid such as air or water experience a drag force that opposes the motion of the object. The magnitude of the drag force increases as the speed increases (unlike the kinetic friction force!).

Empirically it is typically found that

$F_d = b v^n$

where the coefficient $b$ is a constant, depends mainly on crossing area. $n$ is typically 1 – 2.
The terminal speed $v_T$ is the speed at which the drag force $b v^n$ exactly balances the force of gravity $mg$.

$$b v_T^n = mg$$

$$v_T = \left( \frac{mg}{b} \right)^{1/n}$$

A sky diver jumps out of an airplane at 5000m altitude. She reaches terminal speed (due to the drag of the air) after about six seconds.

If a box of steel parts that has the same weight as the diver is dropped simultaneously, the box will fall:

- faster than the diver
- slower than the diver
- the same as the diver

The force of gravity is proportional to the mass of an object, and the drag coefficient is proportional to the cross-section of the object. The box of steel is much more dense than a human body (whose density is about that of water), so the cross-section (volume too) of the box is much smaller compared to that for the person.

In other words, the terminal velocity $v_T = (mg/b)^{1/n}$, which is larger for the box of steel than it is for the person because of $b$. 