A projectile is fired at an angle of 45° above the horizontal. If air resistance is neglected, the line in the graph that best represents the horizontal displacement of the projectile as a function of travel time is

- A. 1
- B. 2
- C. 3
- D. 4
- E. None of these is correct.

A ball is thrown horizontally from a cliff with a velocity $v_0$. A graph of the acceleration of the ball versus the distance fallen could be represented by curve +

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5
A ball is thrown horizontally from a cliff with a velocity $v_0$. A graph of the acceleration of the ball versus the distance fallen could be represented by curve

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

A golfer drives her ball from the tee down the fairway in a high arcing shot. When the ball is at the highest point of its flight,

- A. its velocity and acceleration are both zero.
- B. its velocity is zero but its acceleration is nonzero.
- C. its velocity is nonzero but its acceleration is zero.
- D. its velocity and acceleration are both nonzero.
- E. Insufficient information is given to answer correctly.

Projectile motion:

Typical 3D $\rightarrow$ 2D example;
X-Y independent motions;
X: constant speed;
Y: constant acceleration (free fall).

$$T = \frac{2v_0\sin\theta}{g}$$
$$R = \frac{v_0^2\sin(2\theta)}{g}$$
Problem 3-84.
A projectile is fired into the air from the top of a 200-m cliff above a valley. Its initial velocity is 60 m/s at 60° above the horizontal. Where does the projectile land? (Ignore air resistance.)

Find the time when projectile hits ground:

Projectile elevation \( y(t) = h_0 + v_0 \sin \theta \cdot t - \frac{1}{2} gt^2 \).

Find the time when \( y(t) = 0 \):

Positive solution is:

Horizontal position at time \( t^* : x = v_0 \cos \theta \cdot t^* \).

Example 3-10: ranger and monkey

Ranger aims at monkey, and the monkey lets go of branch at the same time that the ranger shoots. Assume the dart comes out fast enough to reach the monkey while it is in the air.

Does the ranger hit the monkey?
(a) Yes
(b) No
(c) It depends on \( v, h, x, \text{and luck} \).

Problem 3-75 – hitting the monkey

What is the minimum initial speed of the dart if it is to hit the monkey before the monkey hits the ground?

Monkey is \( d=11.2 \) m above the ground; \( x=50 \) m, \( h=10 \) m.

Note that \( \tan \theta = h/x \).

Amount of time it takes for the monkey to fall on the ground is

\[ \Delta t = \sqrt{\frac{2d}{9.8}} = \sqrt{\frac{2 \times 11.2}{9.8}} = 1.5 \text{ s} \]

Because dart must move a distance \( x \) horizontally in this amount of time, we need

\[ x v_0 \cos \theta \Delta t \leq \frac{h^2 + x^2}{(1.5)^2} = 34 \text{ m/s} \]

Example 3-10: ranger and monkey

If the dart is fast enough (before the money lands and runs away), then there will be one time \( T \), at which the horizontal position of the dart is the same as that of the monkey, and you find that the vertical positions of the dart and the monkey (at that time) are the same. So...
Acceleration for a general curved path

Decomposed into:
\[ \mathbf{a} = \mathbf{a}_t + \mathbf{a}_c \]
the tangential acceleration: \( \mathbf{a}_t = \frac{dv}{dt} \)
and centripetal acceleration: \( \mathbf{a}_c \)

Velocity: \( \mathbf{v} \) change rate
Accelration: \( \mathbf{a} \) change rate

Centripetal acceleration:
\[ \frac{v\dot{r}}{r} = \frac{dv}{dr} \frac{dr}{dt} \]
Thus,
\[ \mathbf{a}_c = \frac{v^2}{r} \]
Directing toward center!

Uniform circular motion

Motion in a circle with:
- Constant radius \( R \)
- Constant speed \( |\mathbf{v}| \)
- Velocity is NOT constant (direction is changing)
- There is acceleration (centripetal, always pointing to the center).
Polar coordinates vs. Cartesian

The arc length $s$ (distance along the circumference) is related to the angle via:

$s = R\theta$, where $\theta$ is the angular displacement.

The units of $\theta$ are radians.

$x$-relations:
- $x = R\cos \theta$
- $y = R\sin \theta$

For one complete revolution:
- $2\pi R = R \theta_{\text{complete}}$
- $2\pi = \theta_{\text{complete}}$
- $1$ revolution $= 2\pi$ radians

Speed in Polar coordinates

In Cartesian coordinates, we say velocity $\frac{dx}{dt} = v_x$.

$\rightarrow x = v_x t$

In polar coordinates, angular velocity $\frac{d\theta}{dt} = \omega$.

$\rightarrow \theta = \omega t$

$\omega$ has units of radians/second.

Distance traveled by particle $s = vt$.

Since $s = R\theta = R\omega t$, we have

$v = R\omega$.

The time for each revolution (period):

$T = \frac{2\pi}{\omega} = \frac{2\pi R}{v}$

The number of revolutions per unit time (frequency):

$f = \frac{1}{T} = \frac{\omega}{2\pi}$

Acceleration for uniform circular motion.

Initial velocity has magnitude $v$ and points due east.

Final velocity has same magnitude $v$ and points due north.

Velocity has changed $\rightarrow$ particle is accelerating!

Acceleration constant in magnitude, direction changing with time.

average acceleration $= \frac{\Delta v}{\Delta t}$

the tangential acceleration: $a_t = \frac{dv}{dt} = 0$

the centripetal acceleration: $a_c = \frac{v^2}{R}$

Example: Merry-go-around

Jack stands on a merry-go-around, which makes one turn every 12.0 s, uniformly. He is 2.00 m away from the center.

Find:
1. Jack’s velocity of motion.
2. Jack’s acceleration.

Solution:
1. The velocity is tangential (const speed):

$v = \omega R = \frac{2\pi}{12} 2 = 1.05 \text{ m/s}$

2. The acceleration is centripetal (const in magnitude):

$a_c = \frac{v^2}{R} = \frac{1.05^2}{2} = 0.548 \text{ m/s}^2$
Non-uniform circular motion

An object undergoes circular motion when it is always a constant distance from a fixed point.

Ex. 3-11, A swinging pendulum; fig 3-22

Along a circular path, the velocity is always changing direction, so circular motion involves acceleration (whether or not the speed is changing).

Non-uniform circular motion: - a simple example

An object undergoes circular pendulum motion attached to a 1 m long string. When it reaches the bottom, its speed is 2.0 m/s. In 0.1 s, its speed becomes 1.8 m/s. What is the average acceleration?

Sol.: The velocity changes both magnitude and direction. Decompose to tangential and centripetal:

\[ \alpha_t = \frac{dv}{dt} = \frac{(1.8-2.0)}{0.1} = -2.0 \text{ m/s}^2 \]

\[ \alpha_c = \frac{v^2}{r} = \frac{1.9^2}{1} = 3.6 \text{ m/s}^2 \]

\[ \alpha = \sqrt{\alpha_t^2 + \alpha_c^2} = 4.1 \text{ m/s}^2 \]

Prob. 3-7

The velocity of a particle is directed towards the east while the acceleration is directed toward the northwest, as shown. The particle is:

(a) speeding up and turning toward the north
(b) speeding up and turning toward the south
(c) slowing down and turning toward the north
(d) slowing down and turning toward the south
(e) maintaining constant speed and turning toward the south
Initial and final velocities of a particle are as shown. What is the direction of the average acceleration?

- Mostly up
- Mostly down

The average acceleration is in the direction of \( \vec{v}_f - \vec{v}_i \).