Chapter 9: Rotation

Rotational kinematics, Rotational energy, Moment of Inertia

About Midterm Exam 2

- When and where
  - Thurs March 24th 5:45-7:00 pm
  - Rooms: Same as Exam I, See course webpage.
  - Your TA will give a brief review during the discussion session.
- Coverage: Chapt 5 – 8 (4 chapters)
- Format
  - Closed book, 20 multiple-choices questions (format as in practice exams)
  - 1 page 8x11 formula sheet allowed, must be self prepared, no photo copying/download-printing of solutions, lecture slides, etc.
  - Bring a calculator (but no lap-top computer). Only basic calculation functionality can be used. Bring a 2B pencil for Scantron.
  - Fill in your ID and section #!
- Special requests:
  - If different from Exam I, email me at than@hep.wisc.edu
  - One alternative exam: 3:30pm – 4:45pm, Thurs Mar. 24, Cham 5280 (as before).

3/22/11

The wheels on your 1965 Corvette Stingray have a larger radius than the wheels on your friends 1986 Chevette Hatchback. At some point, your car and your friend are side by side and have the same acceleration. Which car’s wheels have the greater angular acceleration?

1. Corvette
2. Chevette
3. Same

CORRECT

The Chevette’s wheels have a smaller circumference than the Corvette’s wheels therefore the wheels of the Chevette must make more rotations to keep up.

3/22/11

Rotational kinematics

\[ \theta = \frac{s}{R} \]

\[ \omega = \frac{d\theta}{dt} \]

\[ \alpha = \frac{d\omega}{dt} \]

(counterclock-wise +)

\[ v = \frac{\Delta s}{\Delta t} \]

\[ \omega = \frac{\Delta \theta}{\Delta t} \]

\[ a = \frac{\Delta \omega}{\Delta t} \]

\[ R \omega \]

\[ R \alpha \]

3/22/11

Vector Nature of Angular Quantities

The direction matters

- Right hand rule
  - Grasp the axis of rotation with your right hand
  - Wrap your fingers in the direction of rotation
  - Your thumb points in the direction of \( \omega \)

3/22/11
Angular linear:

\[ \alpha = \frac{d^2 \theta}{dt^2} = \text{const.} \quad a = \frac{d^2 x}{dt^2} = \text{const.} \]

\[ \omega = \frac{d\theta}{dt} = \omega_0 + at \quad v = \frac{dx}{dt} = v_0 + at \]

\[ \theta = \theta_0 + ut + \frac{at^2}{2} \quad x = x_0 + vt + \frac{at^2}{2} \]

\[ \omega^2 = \omega_0^2 + 2a \theta \quad v^2 = v_0^2 + 2a x \]

And for a point at a distance \( R \) from the rotation axis:

\[ s = R \theta, \quad v_t = R \omega, \quad a_t = R \alpha \quad a_c = \frac{v_t^2}{R} = R \omega^2 \]

\[ \omega = \frac{d\theta}{dt} = \omega_0 + \alpha t \quad v = \frac{dx}{dt} = v_0 + at \]

\[ \omega^2 = \omega_0^2 + 2\alpha \theta \]

\[ \theta = \theta_0 + \omega t + \frac{\alpha t^2}{2} \]

\[ x = x_0 + vt + \frac{at^2}{2} \]

\[ v_t^2 = v_0^2 + 2a x \]

The tangential component of the acceleration is due to changing speed.

The centripetal component of the acceleration is due to changing direction.

Total acceleration can be found from these components.

\[ a = \sqrt{a_t^2 + a_c^2} \]

You and a friend are playing on the merry-go-round. You stand at the outer edge of the merry-go-round and your friend stands halfway between the outer edge and the center. Assume the rotation rate of the merry-go-round is constant.

Who has the greater angular velocity?

1. You do
2. Your friend does
3. Same

Who has the greater tangential velocity?

1. You do
2. Your friend does
3. Same

Who has the greater centripetal acceleration?

1. You do
2. Your friend does
3. Same

You both cover the same angle in the same amount of time.

for a given angular speed, the tangential speed is directly proportional to the radius centripetal acceleration = \( ru^2 \) - you have a larger radius so therefore you have a larger centripetal acceleration

Rotational kinetic energy, Moment of Inertia:

\[ K = \frac{1}{2} \sum m_i v_i^2 = \frac{1}{2} \sum (m_i r_i^2 \omega^2) = \frac{1}{2} \left( \sum m_i r_i^2 \right) \omega^2 \]

\[ I = \sum m_i r_i^2 \]

MOMENT OF INERTIA DEFINED
Si units are kg \( \cdot \) m\(^2\)

\[ K = \frac{1}{2} I \omega^2 \]

KINETIC ENERGY OF ROTATING OBJECT

\( I \) is like \( M \), the “rotational inertia”.

\[ I = \sum m_i r_i^2 \]
The picture below shows two different dumbbell shaped objects. Object A has two balls of mass \( m \) separated by a distance \( 2L \), and object B has two balls of mass \( 2m \) separated by a distance \( L \). Which of the objects has the largest moment of inertia for rotations around x-axis?

A. A does.
B. B does.
C. They have the same moment of inertia

\[ \text{Case A: } 2 \times mL^2 = 2mL^2 \]
\[ \text{Case B: } 2 \times 2m \left( \frac{L}{2} \right)^2 = mL^2 \]

A hoop, a solid cylinder, and a solid sphere all have the same mass and radius. Which of them has the largest moment of inertia when they rotate about the axis shown?

A. The hoop.
B. The cylinder.
C. The sphere
D. All have the same moment of inertia

Moment of Inertia of a Uniform Ring

- The hoop is divided into a number of small segments, \( m_i \) ...
- These segments are equidistant from the axis

\[ I = \sum m_i r_i^2 = MR^2 \]

Moments of Inertia

1. Cylindrical shell
2. Thin sphere
3. Solid cylinder
4. Solid sphere
For continuum mass distribution:

\[ I = \int r^2 dm \]

Example: A solid bar, \( M, L \):

\[ I = \int x^2 (M/L) \, dx = \frac{1}{3} (b^3 - a^3) \frac{M}{L} = \frac{1}{3} (b^2 + ab + a^2) M \]

The axis matters!

\[ I = I_{\text{cm}} + Mr^2 \]

PARALLEL AXIS THEOREM

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**Calculation of Moments of Inertia**

**Total Kinetic Energy**

\[ \text{motion of center of mass: } KE_{\text{trans}} = \frac{1}{2} MV_{\text{CM}}^2 \]

\[ \text{rotation about center of mass: } KE_{\text{rot}} = \frac{1}{2} I_{\text{CM}} \]

\[ \text{total KE: } KE_{\text{total}} = \frac{1}{2} MV_{\text{CM}}^2 + \frac{1}{2} I_{\text{CM}} \]

The axis matters!

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**Kinetic Energy:**

Rolling without Slipping

Condition: \( \omega = \frac{V_{\text{CM}}}{R} \)

\[ \text{total KE: } KE_{\text{total}} = \frac{1}{2} MV_{\text{CM}}^2 + \frac{1}{2} I_{\text{CM}} \]

\[ = \frac{1}{2} MV_{\text{CM}}^2 + \frac{1}{2} \frac{M R^2}{2} = \frac{1}{2} MV_{\text{CM}}^2 \left( 1 + \frac{1}{MR^2} \right) \]

Also equivalent to the parallel theorem.

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**Application:**

Rolling without Slipping Down Incline:

Find \( V_{\text{CM}} \)

\[ \Delta KE_{\text{total}} + \Delta PE_g = 0 \]

\[ \Delta PE_g = -Mgh \]

\[ \Delta KE_{\text{rot}} = \frac{1}{2} MV_{\text{CM}}^2 \left( 1 + \frac{1}{MR^2} \right) = Mgh \]

Solve:

\[ V_{\text{CM}} = \sqrt{2gh \left( 1 + \frac{1}{MR^2} \right)} \]

Larger \( I \), smaller \( V_{\text{CM}} \)
Two cylinders of the same size and mass roll down an incline. Cylinder A has most of its mass concentrated at the rim, while cylinder B has most of its mass concentrated at the center. Which reaches the bottom of the incline first?

1. A  
2. B  ← CORRECT  
3. Both reach at the same time.

Cylinder A has higher moment of inertia than cylinder B - therefore, it takes longer to roll down.