Chapter 7
Conservation of Energy (cont’d)

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Mechanical energy is of two forms:
- kinetic (by motion)
- potential (by relative positions)

Both of the ability to do work (work-energy theorem).

Energies in Nature
come with several (known) forms:
Mechanical: Electromagnetic & Chemical (heat);
Atomic or nuclear ...

Example: The simple pendulum

- Suppose we release a mass m from rest a height $h_1$ above its lowest possible elevation. Assuming no friction (air drag):
  - What is the maximum speed of the mass and where does this happen?
  - To what elevation $h_2$ does it rise on the other side?

Recollect: Conservation of Energy

- If only conservative forces are present, the total kinetic plus potential energy of a system is conserved.
  - Mechanical Energy = Potential Energy (U) + Kinetic Energy (K)

Conservative forces interchange $U \leftrightarrow K$ (work done), but $E = K + U$ is a constant.

$\Delta E = \Delta K + \Delta U = 0$
Work-kinetic energy theorem: $\Delta K = W$
thus $\Delta U = -W$, for conservative forces only.

- But, if non-conservative forces act, then energy can be dissipated in other forms (heat, for example)
Example: The simple pendulum

- Total mechanical energy: \( E = \frac{1}{2}mv^2 + mgy \)
  - Initially, \( y = h_1 \) and \( v = 0 \), so \( E = mgh_1 \).
  - Since \( E = mgh_1 \) initially, and energy is conserved, \( E = mgh_1 \) at all times.

\[ \begin{align*}
\text{Example: The simple pendulum} & \\
\text{Total mechanical energy: } E & = \frac{1}{2}mv^2 + mgy \\
\text{– Initially, } y & = h_1 \text{ and } v = 0, \text{ so } E = mgh_1.
\end{align*} \]

Example: The simple pendulum

- Velocity is maximum where potential energy is lowest, at bottom of the swing.

\[ \begin{align*}
\text{So, at } y & = 0, \frac{1}{2}mv^2 = mgh_1 \rightarrow v^2 = 2gh_1 \rightarrow v = \sqrt{2gh_1}
\end{align*} \]

Example: The simple pendulum

- To find maximum elevation on other side, note that maximum is reached when \( v = 0 \). Since \( E = mgh_2 \) and maximum potential energy on right is \( mgh_2 \), \( h_2 = h_1 \). The ball returns to its original height.

\[ \begin{align*}
\text{Example: The simple pendulum} & \\
\text{To find maximum elevation on other side, note that } & \\
\text{maximum is reached when } & \text{maximum is reached when } v = 0. \text{ Since } E = mgh_2, \text{ maximum } & \\
h_2 & = h_1. \text{ The ball returns to its original height.}
\end{align*} \]

Example: Airtrack and Glider

- A glider of mass \( M \) is initially at rest on a horizontal frictionless track. A mass \( m \) is attached to it with a massless string hung over a massless pulley as shown. What is the speed \( v \) of \( M \) after \( m \) has fallen a distance \( d \)?

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\end{align*} \]
Example: Airtrack and Glider

- Kinetic + potential energy is conserved since all forces are conservative.
- Choose initial configuration to have $U = 0$.

\[
\frac{1}{2} (m + M) v^2 = mg d \quad \Rightarrow \quad v = \sqrt{\frac{2mgd}{m + M}}
\]

Problem: Toy car

- A toy car slides on a frictionless track shown below. It starts at rest, drops a height $d$, moves horizontally at speed $v_1$, rises a height $h$, and ends up moving horizontally with speed $v_2$.
- Find $v_1$ and $v_2$.

\[
\Delta K = -\Delta U
\]

\[
\Delta U = -mgd, \; \Delta K = \frac{1}{2} m v_1^2
\]

Solving for the speed:

\[
v_1 = \sqrt{2gd} \quad v_2 = \sqrt{2g(d - h)}
\]

Problem: Toy car

- $K+U$ is conserved, so $\Delta K = -\Delta U$
- When the elevation decreases a distance $D$, $\Delta U = -mgd, \; \Delta K = \frac{1}{2} m v_2^2$.
- Solving for the speed:

\[
v_1 = \sqrt{2gd} \quad v_2 = \sqrt{2g(d - h)}
\]

A projectile of mass $m$ is propelled from ground level with an initial kinetic energy of 450 J. At the exact top of its trajectory, its kinetic energy is 250 J. To what height, in meters, above the starting point does the projectile rise? Assume air resistance is negligible.

A) 500/(mg)
B) 250/(mg)
C) 700/(mg)
D) 200/(mg)
E) 350/(mg)
Question

A box sliding on a horizontal frictionless surface runs into a fixed spring, compressing it to a distance $x_1$ from its relaxed position while momentarily coming to rest.

- If the initial speed of the box were doubled and its mass were halved, what would be the distance $x_2$ that the spring would compress?

\[ A) \ x_2 = x_1 \quad \text{B) } x_2 = x_1 \sqrt{2} \quad \text{C) } x_2 = 2x_1 \]

\[
\frac{1}{2} kv^2 = \frac{1}{2} mv^2, \quad \text{Thus } v = x(k/m)^{1/2} = xv(m/k)^{1/2}
\]

So, $2v$ and $m/2$ increases $x$ by $\sqrt{2}$.

Non-conservative forces:

- If the work done does not depend on the path taken, the force is said to be conservative.
- If the work done does depend on the path taken, the force is said to be non-conservative.
- An example of a non-conservative force is friction. Work done is proportional to the length of the path!
  - The mechanical energy is converted to heat.

Spring pulls on mass: with friction

- Suppose spring pulls on block, but now there is a nonzero coefficient of friction $\mu$ between the block and the floor.
- The total work done on the block is now the sum of the work done by the spring, $W_s$ (same as before), and the work done by the friction $W_f$:
  \[ W_i = W_{cons} + W_f = \Delta K \]

(not related to either kinetic energy or potential energy)

Work-energy theorem now reads: $\ W_{net} = W_{cons} + W_f = \Delta K$

Spring pulls on mass: with friction

- Knowing $W_{net} = W_s + W_f = \Delta K$

\[
W_s = \frac{1}{2} kd^2 \quad W_f = \frac{1}{2} \mu mgd \quad \Delta K = \frac{1}{2} mv_i^2
\]

\[
\frac{1}{2} kd^2 - \mu mgd = \frac{1}{2} mv_i^2 \rightarrow v_i = \sqrt{\frac{k d^2}{m} - 2\mu gd}
\]
Many forms of energy:

Question

Which statement is true?
A. Mechanical energy (U+K) is always conserved
B. Total energy is always conserved
C. Potential energy is always conserved

Energy of one form can be converted to another form, but the total energy remains the same.

Newton’s laws \(\leftrightarrow\) Conservation of energy
\[ F = m \frac{dv}{dt} = m \frac{dv}{dx} v \]
\[ F \, dx = m \, v \, dv \]
When \( F_{nc} = 0 \), then \(-\Delta U = \Delta E_k\)

But Conservation of energy is more broadly applicable than Newton’s laws

- Newton’s laws do not apply to systems that are fast-moving close to the speed of light (where Einstein’s theory of relativity applies) and to very small systems (where quantum mechanics applies), but conservation of energy is always valid.

Conservation laws are the consequence of symmetries:
Energy conservation \(\leftrightarrow\) Time translation invariance.